Comparative Study of Fractal Geometry Patterns in Iranian Garden and Landscape Architecture
Case Study: Tabas Golshan Garden

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Abstract

Problem statement: Today, returning to nature and patterning it seems to be one of the most important needs in the world. Iranian architects have been successful in applying scales and connecting different scales to one another, as well as facing nature around them. Geometry, while being abstract, is the most important language through which the architect creates specific spatial qualities.

Research objectives: This article investigates Golshan Garden by expressing its features (rotation, egalitarianism, introversion, centeredness, symmetry, and multiplicity) in Iranian landscaping, while fractal geometry has features including nonlinearity, irregularity, imperfection and non-Euclidean geometry, chaos, and special rules, representing its naturalistic aspect along with the lack of rest and relation of geometries.

Research method: A descriptive-analytic approach has been employed consisting of two steps. The first step included library studies and reasoning to explain the internal validity of the geometry of Iranian garden. Second, the new relationship between the findings of Tabas Golshan Garden as an example of Iranian garden are interpreted and compared with fractal geometry to explain the external validity.

Conclusion: It is found from the study that if the Iranian garden has fractal features in structural, vegetative, irrigation and functional systems, it can be extended to the whole garden. Tabas Golshan Garden as an example of Iranian garden represents fractal architecture design with all the features of fractal geometry and inspired by the nature as well as benefiting from geometric flexibility, definable and recognizable throughout the design and using a form such as the pentagon found in nature.

Keywords: *Golshan Garden, Fractal Geometry, Naturalism, Iranian Landscaping.*

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Fig. 1. The research process based on literature. Source: authors.

that the Persian carpet is a manifestation of the Iranian garden in which the geometry, ornamentation of the garden components, the color composition of the trees and other constituent features of the garden have been used and modeled (Feizabadi, Ansari & Mirhosseini, 2015, 27). Moreover, Poormand and Keshtkar Ghalati investigated the existential causes of Iranian garden construction and concluded that regular and geometric visual elements have been used in order to provide comfort and enjoyment in the Iranian garden (Poormand & Keshtkar Ghalati, 2011, 51). Kave Bookani (2010) investigated the Iranian garden building and its architectural elements, concluding that Iranian garden is a beautiful combination of water, shade, color, architecture, and a calm geometry . Taherkhani and Hesari investigated the geometric structure of the Iranian garden with emphasis on comparing the Fin Kashan garden and Dolatabad garden of Yazd (Taherkhani & Hesari, 2015). Furthermore, Gholam Reza Na'ima, Hamid Reza Ezmati, Bahare Hosseini Moghaddam, Mohammad Naghizadeh, Tahere Nasr, Donald Newton Wilber, Fateme Heidari, Homa Irani Behbahani, Zohre Fazelzade, Azade Shahcheraghi, Luichi Zangri, Nazika Mandana have carried out different studies on the Iranian gardens

and their features. Sala (2006) has pointed out the similarities between architectural styles and fractal geometry. According to Sala, Baroque style, large scale architectures of Africa and East Asia have more features of fractal geometry (Sala, 2006, 163). Among other studies performed in this area, the following can be mentioned: "Studies on the fractal geometry and its effect on architecture and nature" (Kheirat & Shaterzade, 2018), "The role of natural frames in the formation of frames in the area of architecture from the perspective of fractal geometry" (Tafaghodi Khajavi & Nabi Meibodi, 2017), "Fractal nature and its effect on architecture" (Karami Mofrad, 2017), "Geometric features from the perspective of fractal geometry" (Tabrizi & Yadegari, 2017), "Origins of fractal geometry and holly geometry in architecture with case study of the Dome of Soltaniyeh" (Esmkhani & Haditalab, 2016) and "Nature is the source of inspiration; Fractal and architectural geometry" (Zarean, Yadegari & Khosravivand, 2015). There are also other various studies regarding definition of fractal geometry and its diverse dimensions; however, there are no definite and comprehensive studies on the properties of the Iranian garden geometry - Tabas Golshan Garden - from the perspective of fractal geometry.

This research is innovative in its kind. It can also be the beginning of more comprehensive research into the essence of Euclidean and non-Euclidean geometry and its relation to the needs of Islamic countries in the field of architecture in keeping with contemporary world conditions.

Research methodology

This research used a descriptive-phenomenological methodology in an inferential and interpretive approach. Initial information and data were collected from published documentary studies and written sources. Various studies on the subject under study have utilized analytical-comparative-deductive method in relation to fractal geometric structure. Data matching and expression of similarities and differences were used to arrive at the hypothesis under study and finally the research findings have been presented as a conceptual result and model.

Fractal geometry

The fractal word derived from the Latin word fractus - meaning irregularly broken and crushed stone - was first introduced by Mandelbrot in 1975 (Ostwald & Vaughan, 2016). Fractals are shapes that are by no means regular, unlike Euclidean geometrical shapes. Considering the shapes in nature, it is clear that Euclidean geometry is not capable of explaining the complex and apparently disordered shapes of nature (Ghobadian, 2003, 166-167). This geometry has been discussed by many architects and mathematicians for many years and its use in architecture and artificial geometry has been questioned, violated or defended (Eftekharzade, 2005, 100). Fractal theory is applied to describe complexities in the shapes (Panahi, Karimi Khayavi & Mokhtabad Amrai, 2009, 61). Fractals are shapes which are by no means common or regular. These shapes are first of all, irregular, while the irregularity is similar in all scales, indicating a type of regularity by itself. Fractal objects are seen similar from near and far (Khakzand & Ahmadi, 2007, 35-47).

Characteristics of fractal geometry

- Self-similarity: Self-similarity is one of the features associated with using symmetries in patterns such as the use of pattern elements and components that are repeated at different scales (Vaughan & Ostwald, 2018). Each shape is made of smaller pieces that resemble the original shape; in other words, when one of the small pieces is changed into the scale of the original shape, the same shape is exactly obtained (Fischbein,1987). Henry believes that Iranian geometric decorative patterns follow an alternative principle in which self-similar shapes are recursively broken down into smaller proportional copies of themselves (Henry, 2007, 5) (Figs. 2 & 3). Henry has also studied the interior decoration and geometric structure of Islamic architecture and has found that the complex geometry of Islamic architecture reflects the artists' attempt to express emotions through sophisticated geometric designs that incorporate iteration, weight, tone, scale, and composition.

-Small scale: As it can be observed in the following figure, fractals look similar in every scale. Fractal sets include sub-sets which consist of larger sets. Again, these sets include smaller sub-sets which are also similar to the larger sets, and this feature is called small scale (Balilan Asl, 2010, 88) (Figs. 4 & 5).

- Non-Integer dimension: Fractals have unique dimensions which are defined mathematically. Fractal dimension is a mathematical criterion to determine the degree of texture complexity, called non-integer dimension (Davis, 2008, 2) (Fig. 6).

In the above figure, fractal dimensions are 1.465, since each alternative piece is composed of 5 elements which have become smaller with a proportion of 1/3.

- Iteration: Fractals are formed through an iterative process in which every repetition is formed by the previous result. This feature is called iteration (Figs. 7 & 8).

As it is observed in Diagram3, Koch snowflake is always continuous, but non-differentiable. This curve has an infinite circumference with a limited
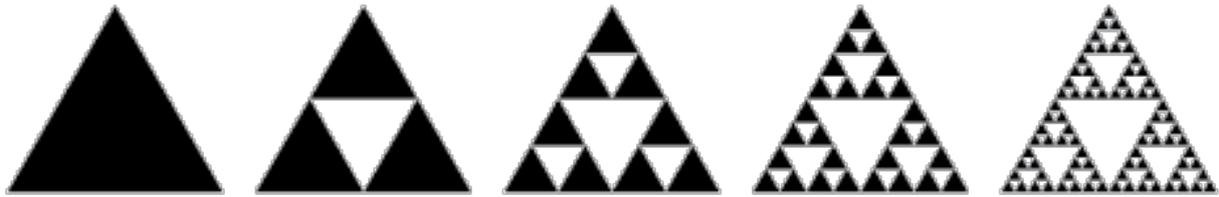
Fig. 2. Development of introversion in Sierpinski Triangle. Source: authors.

Fig. 3. Self-Similar fractal with two different similarity proportions. Source: authors.
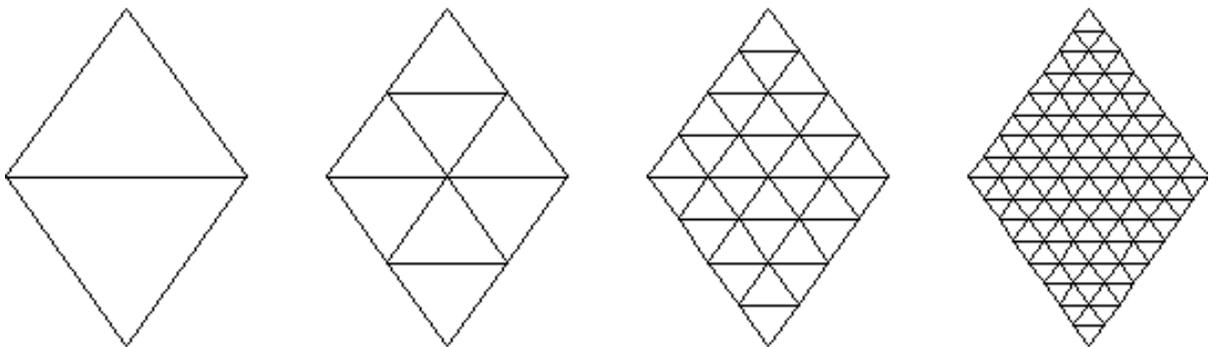
Fig. 4. An example of fractal sub-scale feature. Source: authors.
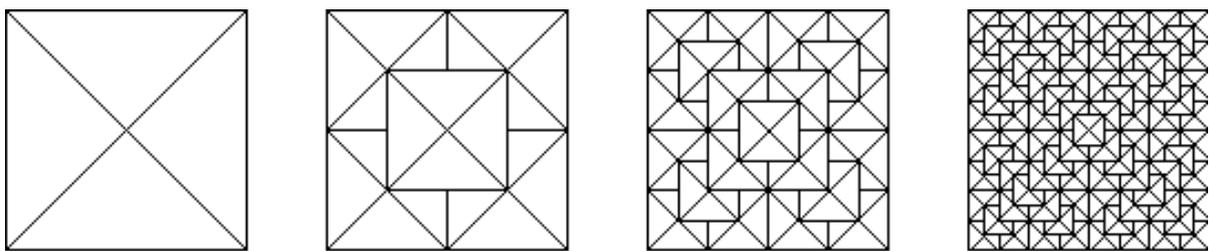
Fig. 5. Fractal sub-scale feature. Source: authors.

Fig. 6. An example of fractal non-integer dimension feature. Source: authors.

area. It is interesting to note that this snowflake has been created through a repetitive and dynamic process, while Euclidean objects are produced by static processes, Sierpinski triangle is also among the most popular fractal objects. This equilateral triangle indicates self-similarity feature (Ghobadian, 2003, 166-167; Davis, 2008, 2-9).

Spatial system and geometry of Iranian garden

Iranian culture does not consider the human separated from nature, but sees him along with other elements of nature to understand as signs of God (Irani Behbahani, 2007). Therefore, Iranian architecture and art seems to be extremely naturalistic.

The Iranian garden is a combination of simple and

harmonious elements, a well-balanced and robust relationship, a distinctive geometrical system, perpendicular lines, rectangular garden beds, and a well-watered wide-open network (Arabi & Vahdati, 2015) (Figs.9, 10 & 11). Referring to the historical origins of this word, Mirfendereski basically calls the Iranian garden Chaharbagh (or four-garden) and says that in the system of establishment, two axes perpendicular to each other divide the garden into four main parts (Mirfendereski, 2005). This is while Daneshdoost considers Iranian garden as a combination of lines and main axes which divide the garden space into four sections (Daneshdoost, 2011).

Generally, if there are no philosophical principles, namely unity, identity, authenticity, honesty, and practical principles of scale, including proportion, infinite regression, composition, and equilibrium that are inextricably linked, the architecture cannot be examined.

Tabas Golshan Garden

Tabas Golshan Garden, the city's most important garden, falls into the category of governmental
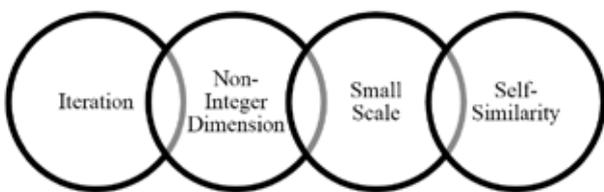
Fig. 7. Different dimensions of fractal geometry. Source: authors.
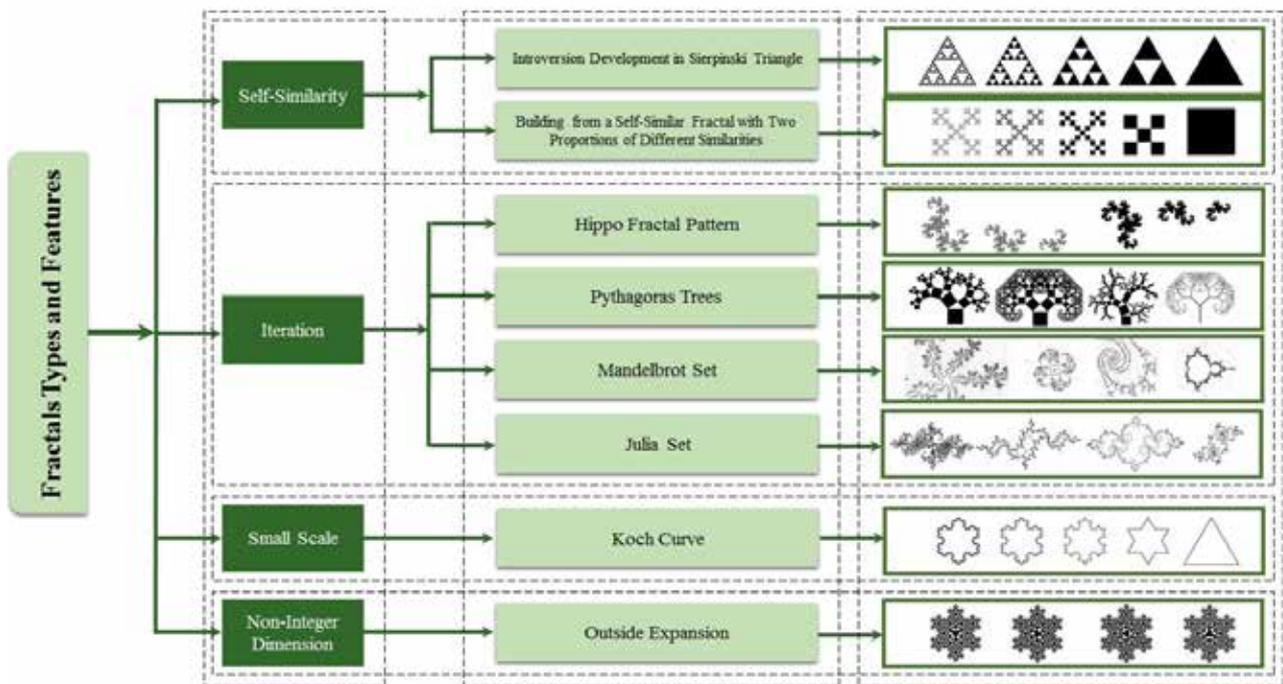
Fig. 8. Categorization of fractal features. Source: authors.
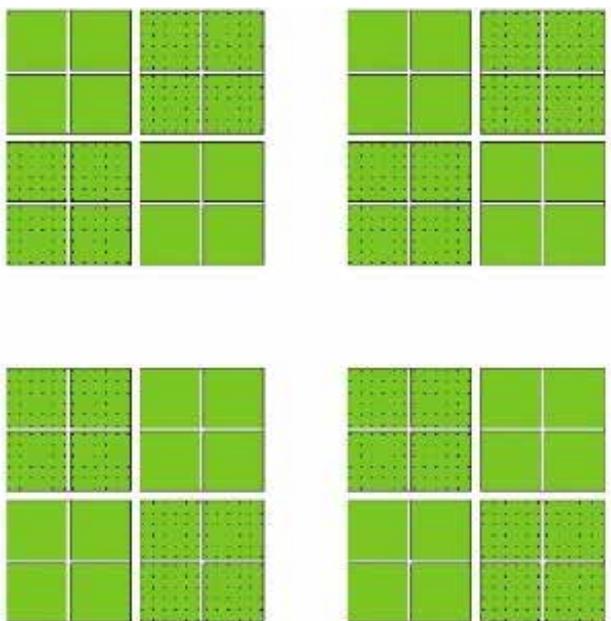
Fig. 9. Layout of Islamic Chaharbagh and smaller gardens. Source: authors.
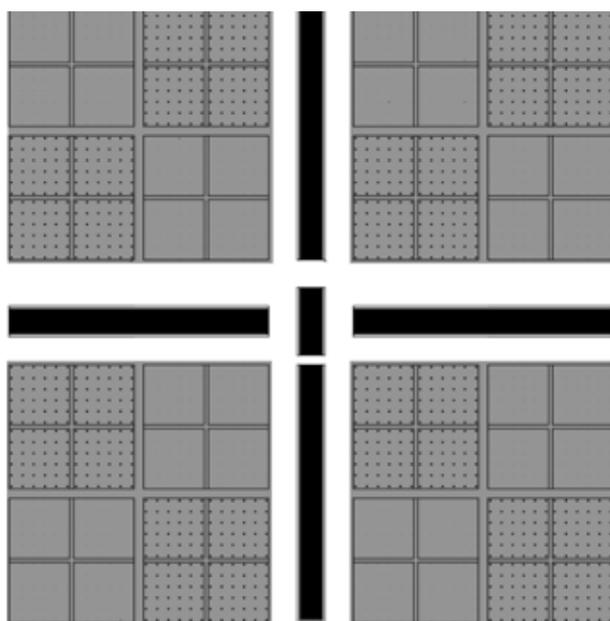
Fig. 10. Quadruple schema derived from ancient beliefs introduced as an ancient Iranian garden pattern. Source: authors.
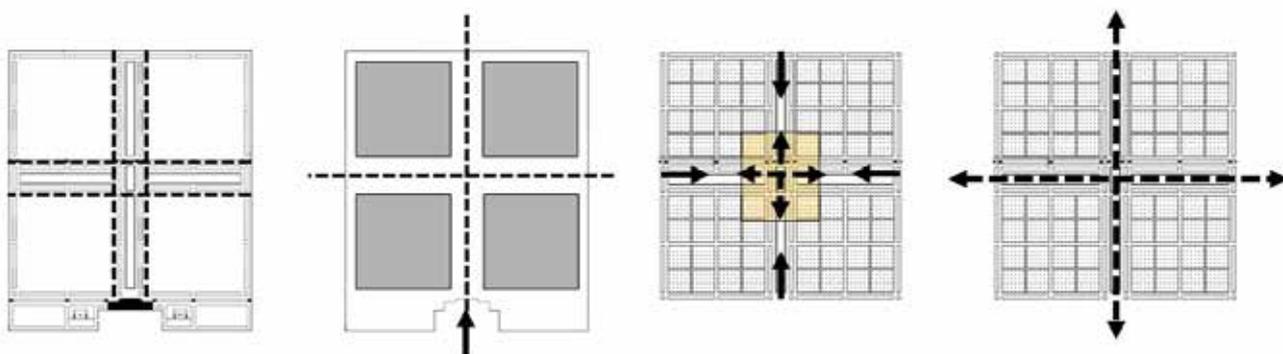
Fig. 11. The main lines and axes that divide the garden space into four sectionS. Source: authors.

gardens and has only a courtyard and no other monuments can be found in its interior. The garden was built by Mir Hassan Khan, the third ruler of Tabas, appointed by Nader Shah. Mir Hassan Khan donated this garden and many other properties to the public and charity affairs in his will in 1803 (Moravej Torbati & Hosseini, 2013, 92).

The garden has a design that is most likely implemented in a palm tree field; the length of the garden from the entrance to the end of the entrance axis is 266 meters, while the two front sides are 266 and the garden's width is 260 meters. The garden has an approximately 74438 square meters of area (Daneshdoost, 1990). The design of Tabas Golshan

Garden is square, with geometrical divisions along the sides of its fence (Fig. 12). The design of Chaharbagh is evident in this architecture. The pattern is a square or rectangular boundary between the garden walls, the surface of which is interspersed with two perpendicular intersections of water in four equal portions, and there is a water pool in the center (Shahcheraghi, 2015, 7) (Fig. 13). However, the use of the word Chaharbagh instead of Charbagh has been reinforced from the Safavid period onwards, which also includes research reports on Tabas Golshan Garden (ibid.). From the architectural point of view, the facade of Golshan Garden is a fairly typical building, making one of the most distinctive

Fig. 12. Aerial image of Golshan Garden location in Tabas.
Source: www.google.com/earth

Fig. 13. Entrance of Golshan Garden. Source: authors.

Fig. 14. Design of Golshan Garden as Chaharbagh. Source: Moravej
Torbati & Hosseini, 2013, 96.

and important buildings in the city (Fig. 14)
(Daneshdoost, 1990).

In the following, the article aims to study the
geometric pattern of Tabas Golshan Garden as
an example of the Iranian garden along with the
Fractal Geometry Pattern, to analyze their common
dimensions such as the structure of the city and the
location of the garden in it, along with the structural
system, irrigation system, as well as the planting
system of the garden.

The most important principle in the design of the
Iranian garden is its structural system, which is
based on the geometrical order. Iranian garden, while
a wall surrounds it, is divided by vertical axes on
the main and secondary axes. In addition to creating
a peripheral composite wall, the construction work
on the Iranian Garden deals with stair-breaking
walls, paved surfaces, streams, ponds, and pools, all
of which follow the geometric order of the garden
and in fact, form the landscape of the garden. The
main garden buildings are on two scales, big and
small. The big scale contributes to the main space
of the garden and the small one defines the garden
surroundings. On a large scale, the facade, staircase
and mansion are usually built on the main axis,
while on the small scale, secondary structures such
as bathrooms and other services are located on the
side walls, whose entrance is sometimes simple and
other times composite (Mirfendereski, 2005).

Architecture of Iranian garden is organic. The term
"organic" represents an architectural concept, not
an essence. The original meaning of this term in
architecture is associated with the link between the
total and the parts (Mahmoudinezhad, 2009).

The structure of Tabas Golshan Garden is
comparable to fractal geometry in all parts of the
garden, including plan, ornamentation, irrigation
and vegetation. The Iranian garden consists of
a combination of water, plants, and buildings.
Fractal-sized plots are found in the overall structure
Golshan Garden plan. One of the main features of
the geometry of the Iranian garden is symmetry and
proportion, which is seen in the vast majority of

them. Tabas Golshan Garden is not an exception and this symmetry as well as proportion can be observed in structural, irrigation, and vegetation dimensions of the geometric plan. The Iranian architecture has been paying special attention to privacy and introversion from the beginning. This introversion has been created in this garden due to the reticulate design of the walls around the garden, the shaded trees and also the mansion. Another characteristic feature of this garden is its multi-layered structure (Fold style), because it can be said that the garden is made up of three layers that are connected by vertical connections. The first layer is at the beginning of the entrance, while the second layer is concentrated on the central pond and the third layer is the highest level at the far end of the garden. In addition, the planting layer, the pedestrian crossing layer and the third layer for the water flow are seen in the formation of Golshan Tabas garden components. In this plan, a micro and macro scale rotation can be observed, so that the intersection of the lines of symmetry makes the center of the plots rotation, and the paths of water flow (Fig. 15).

The decoration of Tabas Golshan Garden contains

Islamic nodes whose structure is self-similar. In addition, the iteration and replication of this structure is of a small scale nature, so that what is sought in details can be found at large scale. The nodes are extremely regular and extendable in addition to possessing a highly sophisticated structure (Fig. 16).

• **Irrigation system**

In Iranian garden water is directed underground and is fountains in special places Water flows everywhere in the garden. Its motion in four directions and streams is same the symbol of four streams in the heaven, applied to the Iranian garden design (Diba & Ansari, 1996). In addition to its functional and vital role in plant irrigation, water plays a prominent role in landscaping and creating pleasant sounds (Mirfendereski, 2005).

Water has the properties of refreshing, moisturizing, cleansing, reflecting and life-giving. Water in Iranian architecture is considered as a central element and the architecture around it shapes places, links spaces, and influences environmental comfort, ritual and fluidity. The presence inside the mansion and the movement between interior and exterior, especially in relation to the garden and the courtyard, as well as the presence
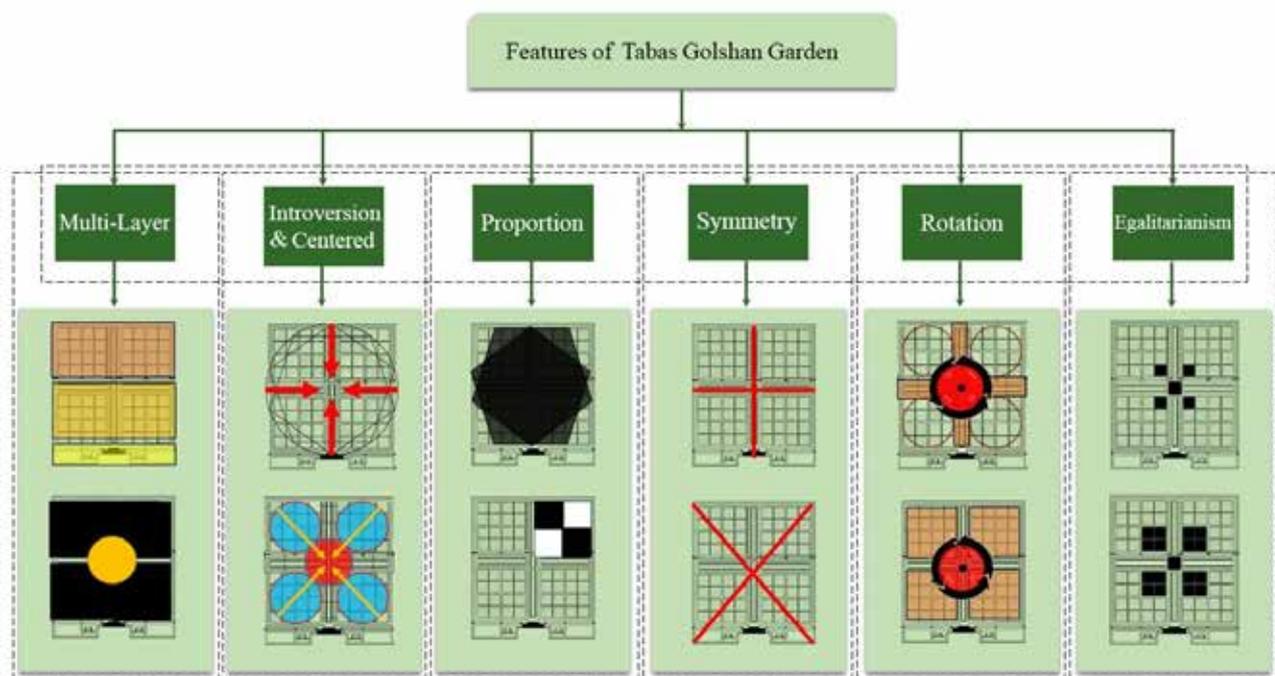

Fig. 15. Comparison of fractal geometry and geometry of Tabas Golshan Garden structural system. Source: authors.
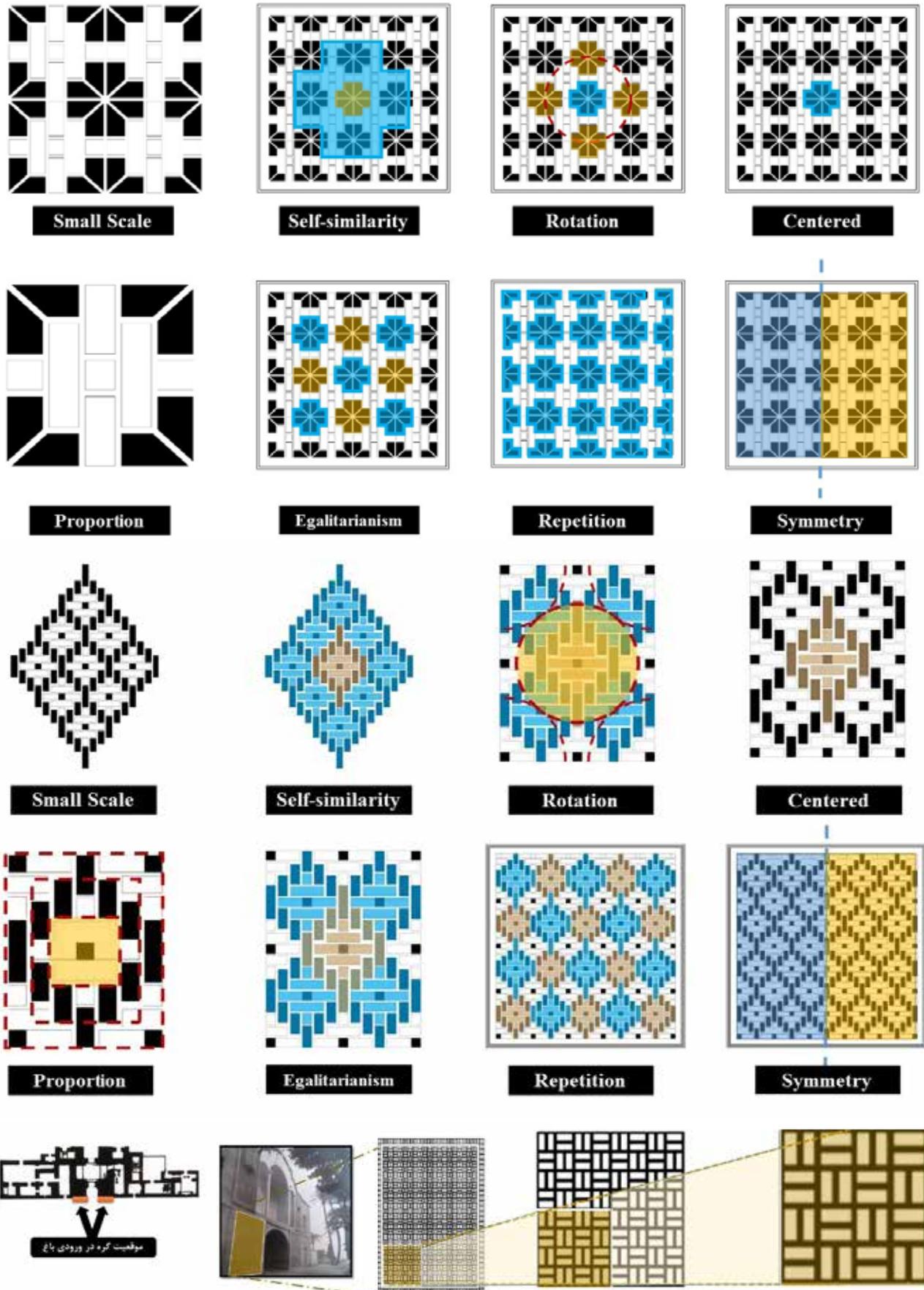
Fig. 16. Comparison of Golshan Garden nodes and fractals from the common geometry perspective. Source: authors.

at the entrances indicate the functional value of aesthetics and belief in water in Iranian architecture (Naghizade & Doroudian, 2008).

The irrigation system in Goshan Garden has a self-similarity feature, so that in the whole irrigation system, water movement paths are larger scales of water resettlement sites and the dimensions of water movement paths are increased relative to their resettlement location. In Small scale features, each set consists of a smaller subset, which are similar to larger sets. In the central fountain of the garden, the water pond and the waterways are observed on either side of the garden, each of which is a representation of the large pond in the middle of the garden, which reflects the small scale geometry of the Golshan Garden irrigation system. Water paths are repeated along the garden, so that there are totally 8 similar paths. Fractal dimension of the irrigation system in Tabas Golshan Garden follows the absolute value of Log4/log^2 in which N=4 and L=1.05, with a final answer of 2 for the equation (Fig. 17).

• **Vegetation system**

In the Iranian garden, the plants have their own order of shading trees on either side of the garden paths, emphasizing the pivotal paths given their importance and relying on the variety of shading trees. These trees are only one type of tree at a time, such as plane trees and at times two species such as plane and cedar trees. In the lateral axes that intersect the main axis orthogonally and form paths between the plots, there are also rows of shading trees (Mirfendereski, 2005).

Rows of trees and buildings as well as intersecting paths that make it possible to exchange can be a fractal. Moreover, the trees with their fractal leaves and branches catch the light and stand against the wind, while the leaves are located in optimal conditions. The rules governing vegetation growth lead to features, existing in small scale and being transferred to larger scales as well (Mahmoudinezhad, 2009).
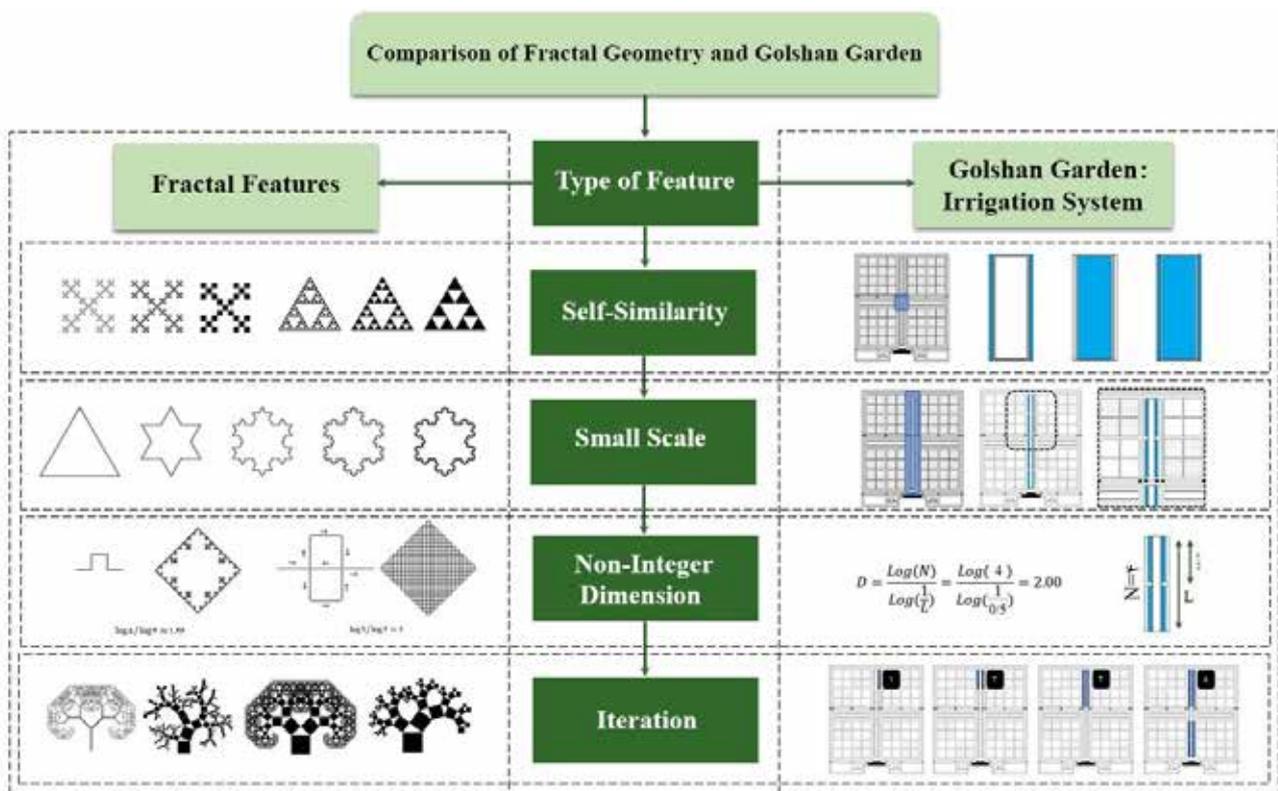
In Tabas Golshan garden, the vegetation and plots

Fig 17. Comparison of fractal and irrigation system geometry of Tabas Golshan Garden. Source: authors.

have self-similarity, so that each plot is a smaller version of the larger plot. As the dimensions of the plots grow, the plots increase in size and are replicated. It can be said that the structure has a hierarchy in addition to the plant system which reflects hierarch in terms of planting and shading features. The fractal non-integer dimension of a self-similar object is calculated as the absolute value of $\log N / \log \left(\frac{1}{L}\right)$, in which $\left(\frac{1}{L}\right)$ represents the length of the reticulate dimension and N indicates the number of cells which measure the object. Regarding the fractal dimension of vegetation system in Tabas Golshan Garden, it should be

noted that according to the calculations for the garden plots in which N=16 and L=1/4, the answer to the equation will be 2, which is equal to non-integer fractal dimension of Peano curve in fractal categorizations (Fig. 18).

Discussion

The main features of buildings constructed on the basis of fractal geometry and patterns are: flexibility to climatic conditions, cultural and functional issues, innovation, producing positive feedback, feeding on negative feedback from natural systems, definable and recognizable in the overall design
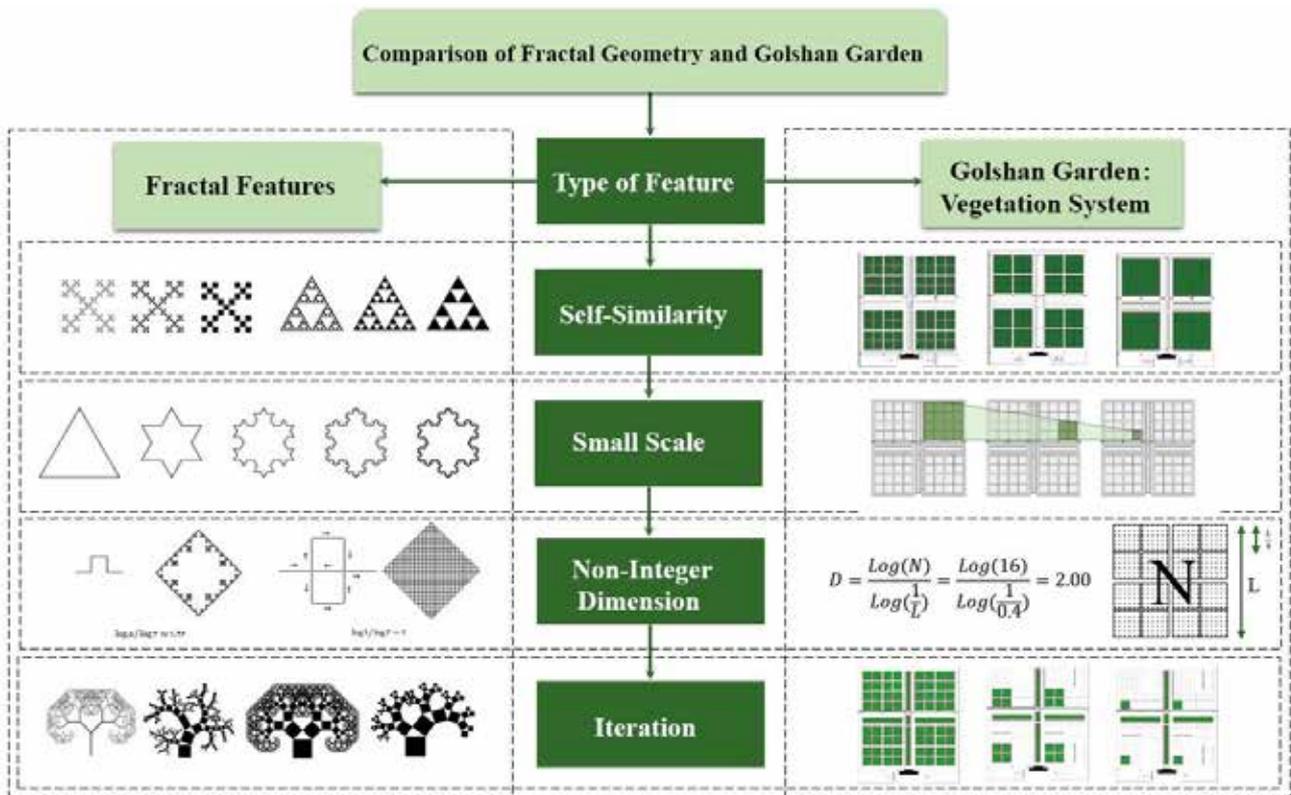

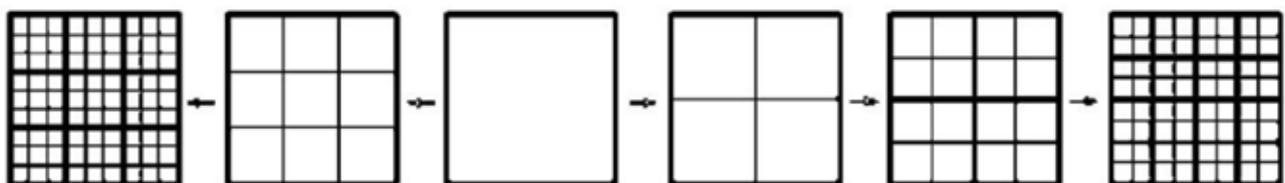Fig. 18. Comparison of fractal and vegetation system geometry of Tabas Golshan Garden. Source: authors.

Fig. 19. An example of fractal form with continuous. Source: authors.

and unpredictable in details, most mediated by the surrounding environment, maximum use of natural elements and textures, paying attention to all scales of design, productive and continuous information taking into account the user's perceptual capacity, the use of symbols, signs and impressions, the ability to create memories and enable the observers to forget (The power of creation and elimination of information) (Mahmoudinezhad, 2009) (Fig. 19).

The art of Iranian gardening has been applied in such a way that the most prominent of these has been the idea of Chaharbagh and the enclosure of the mansion (architecture) with the garden and the prominent geometry. In such a garden, a sacred outline of the walls or the interior of the garden kept everything in balance and marked a rectangular space that was divided into four sections by water canals or corridors. The term "Chaharbagh" taken from Old Persian was used for this design and is still being used in the contemporary Persian (Khansari, Moghtader & Yavari, 2004).

In view of the material and tables presented in this article and the comparative study of the nature, numerous valuable lessons and design guidelines are provided. In other words, it is important to understand the effective and shaping principles of these lines and forms, emphasize on the meaning rather than appearance, and apply the principles that guarantee growth, life of organisms and evolution in the environment. Undoubtedly, using a visual conceptions of nature in architecture, cannot give it meaning and identity. Therefore, the presence of the spirit of nature in architecture give it, manifestation and development.

In fact, the unity of plurality in the architecture of Golshan Garden illustrates its fractal geometry along with its totality from one part to another. The simple fractal form cannot have any added values to today's architecture with a superficial imitation. By knowing and understanding the fractals, it is possible to create quality Iranian gardens that meet the new human needs (Fig. 20).

The Iranian garden, especially Tabas Golshan

Garden, has many concepts that still have much to discuss and can guide us in designing today's gardens, because its principles have been used by many architects and designers in a variety of different ways and approaches. It is hoped that other scholars and researchers will continue to take an effective step in the study of the relationship between the Iranian garden and the fractal geometry in it and continue this path.

From the studies performed, the researchers found the fractal properties as well as the geometry of Golshan Garden, including fractal properties such as nonlinear and non- Euclidean geometry, irregularity, imperfection, Chaos, and special as well as infinite rules. In analyzing the geometry of Golshan Garden, three structural, irrigation and vegetation systems have been discussed, as they have been mentioned as the main elements of the Iranian garden. The structural system of Tabas Golshan Garden has symmetry, rotation, proportion, regeneration, centeredness, multiplicity, conceptualism, and symbolism features in both details and decorations. Also, the results of investigating the irrigation and vegetation systems of Golshan Garden showed the characteristics of symmetry, proportion, multiplicity, symbolism and aesthetics. By comparing the features of fractal geometry and the geometry of Golshan Garden four
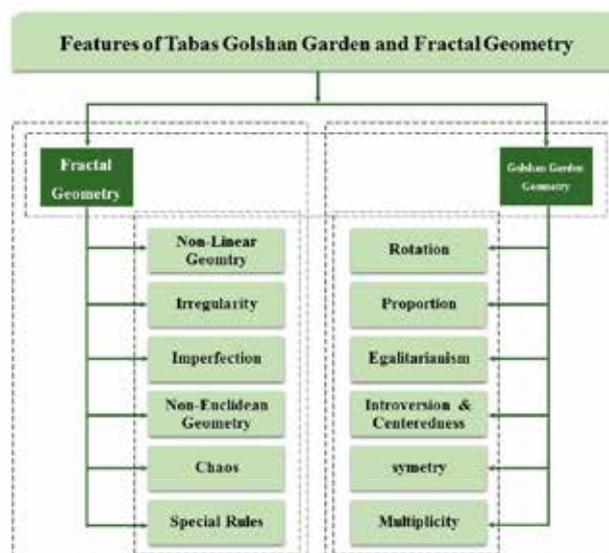

Fig. 20. Features of Tabas Golshan Garden and fractal geometry. Source: authors.

common features of self-similarity, small scale, non-integer dimension, and iteration were found. Studies show that the geometry of Golshan Garden and the fractal geometry have both natural origins. On the other hand, there is balance in nature and the structural, irrigation and vegetation systems of Golshan Garden are also in balance, indicating that its geometry is taken from fractal naturalistic geometry (Fig. 21).

Conclusion

The artist's intention is to transform the hidden geometry of the universe into explicit geometry. In Iranian culture, human and nature does not separate together, due to both are signs and symbols of God. Therefore, Iranian architecture and art appear to be highly naturalistic. In fact, it can be said that the

overall purpose of designing an Iranian garden is to achieve a communion with nature that exists as an attraction in the human essence.

The results show that using features such as self-similarity, small scale, non-integer dimension, and iteration derived from the nature as well as using the form. For example pentagons as frequently seen in the nature and able to provide those features, can be considered as the principles of fractal architecture design in Iranian garden such as Tabas Golshan Garden. Fractal geometry is evident in diverse and self-consistent spaces as well as spatial typology of Golshan Garden. The designers of the Golshan Garden have derived fractals from self-similarities of nature and have found this geometry a way of expressing the whole in detail and the detail in the whole.
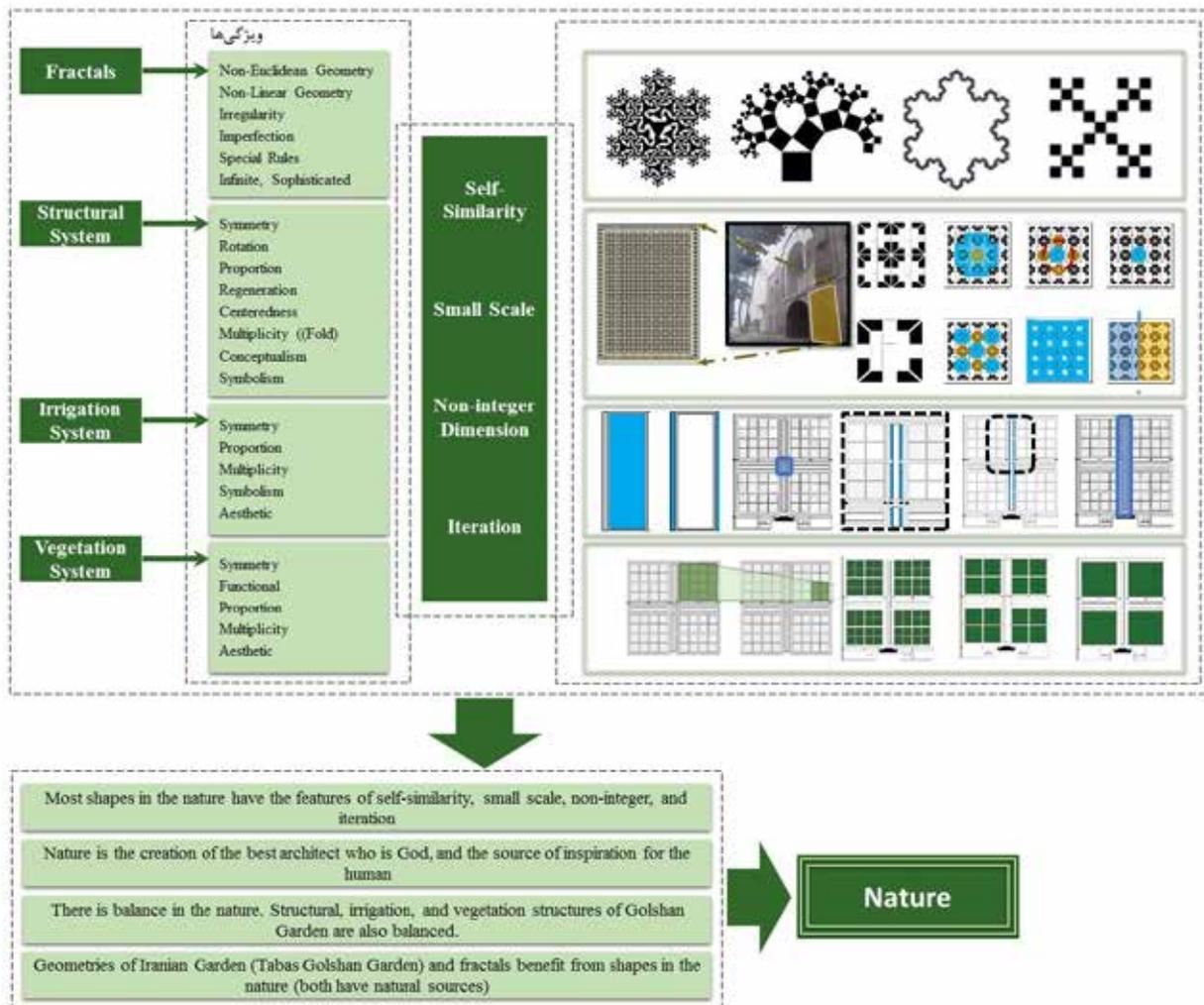

Fig. 21. Features of Tabas Golshan Garden and fractal geometry. Source: authors.

In general, understanding the nature, essence and operation of a compound leads to unity. To achieve this unity and to obtain a single overall result, it is essential to know the components and integrate them correctly into the architectural requirements that have been best demonstrated in the systems forming Tabas Golshan Garden. In this way, the clarity of the architect's concept and purpose is essential, because full awareness of the concept of work that must be done and accurate understanding of the purpose is the basis of the architectural work to achieve the desired unity.

In conclusion, it can be said that:

1. The components of Tabas Golshan Garden geometry indicate the features of fractal geometry, including self-similarity, small scale, non-integer dimension, and iteration (See Fig. 21).

2. There are patterns of fractal geometry in Iranian garden if its structural, irrigation, vegetation and functional systems are derived from fractal geometry.

3. Tabas Golshan Garden reflects features of fractal geometry in terms of the features of its constituent systems.

4. The space-time approach is a four-dimensional continuum that combines three dimensions of space with time dimension. Tabas Golshan Garden has time continuity in this regard. In other words, it can be said that this garden represents stability in obtaining fractal features in the fourth dimension of its nature.

5. Tabas Golshan Garden indicates features of flexibility against climatic conditions, cultural and functional issues. In this garden, along with being innovative, identifiable, and understandable in plan, while the details with maximum use of natural elements, materials and textures, are unpredictable. In Tabas Golshan Garden all features such as: attention to all designing scales, use of symbols, signs, and indications, and the ability to create memories, are reflected in fractal geometry.

6. Beyond the unique geometry of Tabas Golshan Garden in different dimensions, other concepts and features such as conceptualism, symbolism, and multiplicity are also observed which need more investigations.

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