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## Explanation of Fractal Geometry Laws in the Structural Form of Architecture Presentation of a Form-Based Architecture Model\*

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### Abstract

**Problem statement:** From the perspective of modern sciences, fractal knowledge is a study of self-similar and self-organized structures that are the conceptual core of understanding nature's geometry. From a mathematical viewpoint, nature follows fractal rules to create different forms. In this rationale, a direct interaction between architecture and fractals can be proposed.

**Research objective:** This research, by posing questions about how fractal geometry is applied, explains the geometric laws of nature in the process of self-organizing in form-based architecture.

**Research method:** This applied research employs the method of correlation in applying geometries according to fractal properties conducting a qualitative, phenomenological view. In this study, seven stages are explained in fractal geometry as a form-based structure. These stages in form-finding, the hierarchy of connections, and geometric progressions provide a credible process in the application of fractal geometry to architecture. In addition, an architecture inspired by principles of nature's geometry is based on three laws and functions in structure as described by the research: the law of self-similarity, the law of self-organization, and the law of unity. To validate these laws, three buildings are selected from the Gothic, Art Nouveau, and traditional Persian architecture periods, according to fractal properties. Fractal rules are analyzed in the evaluation of these structures using a comparative-descriptive model.

**Conclusion:** Based on the results, it can be concluded that fractal geometry has been incorporated into the structure, elements composition, and aesthetic concepts of ornamentation in the architecture of these edifices. Inspired by nature's geometry, laws of proportions, rhythm, and self-similar forms, these architectural structures have created a wide range of fractal patterns, generating internal connections and external unification among the structural elements and architectural forms.

**Keywords:** *Self-Similarity, Self-Organization, Unity and Plurality, Structural Form, Fractal Geometry.*

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## Introduction

As a novel view in the real geometry of nature proposed by French mathematician Benoit Mandelbrot, fractal science studies the geometrical aspects of natural structures. In his theory, Mandelbrot explained that the universe has fractal dimensions and natural structures have fractal properties (Mandelbrot, 1983). His idea in mathematics states that nature follows nonlinear geometry and fractal rules to create various shapes. With respect to self-similarity and self-organization properties in fractals, it can be proposed that fractal geometry is a novel paradigm whose greatest potential is the ability to create form-based structures. In this regard, a direct relationship between architecture and fractal can be addressed. While architects and thinkers stated theories on explaining fractals and their patterns, a comprehensive explanation of nature's geometry and the relevant laws in architecture has been considered in this research. This study raises questions about: How can nature's geometry be applied in the process of organizing the structural form of architecture? How can the function of fractal geometry in elements composition, the hierarchy of connections, and the aesthetics concept of ornamentation in architecture be explained? The main objectives of this research are 1. Discovering and presenting the structural idea in architecture by examining fractal geometric structures 2. Description of the composition of elements and hierarchy of connections in creating architectural spaces by studying fractal functions 3. Assessment of fractal qualities of ornamentation in architecture based on comprehensive fractal patterns. In addition, a description of applied principles in nature-inspired engineering systems is provided to enable a scientific-applied method of enhancing the quality of architecture.

## Research hypotheses

**Hypothesis 1:** To answer the first question about the application of fractal geometry in the structural form of architecture, an exploration is carried out on a sample structure in evergreen trees. Through a study of different longitudinal and transverse

sections of the sample, geometric stages of fractal structures are adopted and applications of linear and nonlinear geometry in the structure of the sample are explained. This method can describe the self-organization in natural structures and enable the application of fractal forms into architecture.

**Hypothesis 2:** Along with answering the second question, this research states that the function of fractal geometry in elements composition, the hierarchy of connections, and aesthetics concepts at different levels are based on the laws of fractal geometry. Derived from the process of self-organizing in fractal structure in hypothesis 1, these laws are developed with respect to fractal qualities of the selected buildings.

## Literature review

Within the phenomenology of nature's geometry and fractal, an ontological view influenced by novel cosmological sciences ultimately leads to the manifestation of various visual effects and introduces a wide range of applications in architecture. In the seminal theories and with respect to the laws of cosmology, Albert Einstein believed in the discipline governing the universe. Along with the same belief, French mathematician Henri Poincaré proposed a new path for the mathematical order of the universe, which led to the development of chaos theory. In this way, the scholars of cosmology presented an ontology-oriented science within the framework of chaotic systems that represent the order in disorder. The ideas proposed in chaos theory was the development of a dynamic and advanced system representing a high level of organization in natural systems. "The balance between order and disorder in chaos theory can be a suitable solution to the stability of architectural spaces" (Ashrafi, 2017). In these systems, the discipline of nature originates from fractal geometry patterns that illustrate the laws of geometry in the structure of natural phenomena. In this regard, Mandelbrot adapts fractal nature with nonlinear geometry in form and structure, which is one of the most important indices of similarity, continuity, rhythm, and harmony (Mandelbrot, 1983). In other words, geometrical fractal patterns are a

novel approach to the perception of the natural order in the modern ontological philosophy. In line with the perception of nature's system, architects like Ruskin (1857), Alberti (1987), and Antoni Gaudí pioneered the application of the law of proportions, natural forms, and similar structures in structural stability, function, and the aesthetics derived from the natural structures in architecture. In the research conducted in the recent decades on the application of fractal geometry, another group of architects such as Bovill (2000), Crompton (2004), Sala (2006), Joye (2007), Haghani (2009) and Ramzy (2015) examined the idea of fractals in the structure, composition of elements, and ornamentation in the buildings of the Gothic period. This group introduced fractal branches in capitals, continuing self-organizing in a hierarchy of connections, self-similar geometries in ornamentation, and creating rhythm in structural form as a combination of fractal processes in these buildings. In converging theories with the first two groups, another group of architects like Jencks (1997; 2002; 2011), Mousavi (2009), and Nogherekar (2013) reviewed systemic architecture from the fractal perspective. In their view, fractal architecture provides an artistic interpretation of the physical reality of the universe that establishes a precise link between technical requirements and aesthetical concepts in architecture based on creative and self-organization systems. Jencks states that "human perception of the world is reflected in architecture" (Rahmani, Etessam & Mokhtabad, 2017) and "form follows worldview" (Jencks, 1997; 2011). In expressing the form, Mousavi believes in meta-functional architecture based on natural systems. Mousavi considers form as a particular function influenced by different variables of nature's order, mathematics, and harmony, which expresses novel functions in architecture (Mousavi, 2009). Although these theorists and architects deal with different aspects of applying nature's geometry and fractal patterns in architecture, a description of how to apply fractal geometry in architecture is yet to come. This research explains the geometry-oriented stages of fractal structure and defines comprehensive applied laws for this stable system in the structural form of architecture.

## Theoretical foundations of the research

### • Perception of nature's order and fractal geometry

The systemic method is one of the most comprehensive attitudes in applying nature's geometry and fractals in architecture. In this rationale, the properties of fractal geometry are examined in a correlation-based system to understand the process of development, evolution, and the principles governing this geometry. The main purpose is to recognize the laws of nature and to utilize fractal geometry strategies in architecture structure and form. In this regard, fractal properties examined in three fundamental subsystems: 1. The structural system with an emphasis on sustainability aspects, 2. The functional system with an emphasis on structural aspects and 3. The physical system emphasizing the unifying aspects of components.

#### - The structural system

The structural system consists of the firmness actions in the structure of nature that shapes through geometric interchanges in the main elements to serve as an important mediator in how form and function relate. This level of harmony between form and structure is required to achieve sustainability goals (Nogherekar, 2013). The principles of "connectivity" and "branching" are important elements of the structural system. In the principle of connection, nature uses a mediator element to create connections for the best response to the structural system in the next step. In branching or mutation, a more transcendental manifestation of intelligence and purposefulness takes shape through an evolutionary process (ibid.). The universe is productive and evolving, "always creating variable goals and exposed to a distinguished momentary quality" (Taghvaei, 2012). As Jencks notes, the mutable, momentary, and nonlinear orders introduce a new narrative in architecture based on fractal forms that accompany self-similarity in structure (Jencks, 2011).

#### - The functional system

The functional system incorporates natural actions in creating a hierarchy of connections, ways of loading technique, and timely structural behavior. Each tree finds the shape of trunk and branches in resistance to

environmental forces and is sustain to compressive and tensile forces (Noghrekar, 2013) so instead of a straight line, it curves to coordinate its form and function in an organized way. The branches and the balance in their organization are important in creating structural connections. The principles of “continuity” and “consistency” are essential for a functional system, and create an interaction which is necessary for establishing hierarchical connections. In the principle of continuity, In the principle of continuity, nature, with a sequence of similar elements creates rhythm in the process of evolution to respond to the structural organizing system in the best way. “The new patterns of order are consistent with self-organization and the regenerating evolutionary processes” (Taghvaei, 2012). In the principle of consistency, nature uses functions of nonlinear geometry as parabolas and hyperbolas to make structural connections.

#### - The physical system

The physical system is the law of proportion and the conformity of all components into one unified being. Natural systems have a property of unity in diversity, in the sense that multiple, diverse elements collaborate to exhibit a form of unity (Noghrekar, 2013). The principle of “harmony and unity” is an important nature of the physical system, in which natural structures in harmony move toward unity and re-originate from this unity. In examining patterns of nature’s geometry, Mandelbrot points out the important principle of homogeneous repetition from component to whole, in that the whole is very similar to the component, and each set of components is made up of the whole (Mandelbrot, 1983).

#### • Structural properties of nature’s geometry and fractal

##### - Self-similar elements

Historically, it can be stated that the theories of architecture in the past were often inspired by nature. John Ruskin, a thinker and philosopher of aesthetics, is the first to describe and illustrate self-similar structures (Fig. 1-A). Ruskin appreciated nature’s forms and encouraged architects to apply the laws of nature in their works. In his view, “continuity” in nature is the result of “self-similar

sequences”, and stated that “Another important and pleasurable way of expressing unity is by giving some orderly succession to a number of objects more or less similar. And this succession is most interesting when it is connected with some gradual change in the aspect or character of the objects” (Ruskin, 1857, 259).

Mandelbrot explains natural phenomena have fractal properties and patterns. Geometrically, a fractal is a combined self-similar form that has a unified arrangement, for example, Mandelbrot’s fractal tree (Fig. 1-B) is a structure consisting of components that enlarged in proportion to each other. In other words, a fractal is a structure that every component is self-similar to the whole (Mandelbrot, 1983). Corresponding to the fractal self-similarity property and the method of applying Persian geometry in the Yazdi Bandi structure (Fig. 1-C), it can be stated that self-similarity has been created in different proportions and scales in this type of structure. Persian geometric patterns use the principle of geometrical interchanges, in which self-similar elements are transformed into proportional components to form a geometric hierarchy of connections and branching in the structure.

##### - Hierarchy of connections

Spanish architect, Antoni Gaudí, pioneered the use of natural forms incorporating hierarchical connections in architecture. In Gaudí’s view, there is no straight line in nature; therefore he used nonlinear geometry as a primary reference of architecture. He stated that nature generates extraordinary structures that architects must perceive and understand. By studying natural forms, Gaudí incorporated the function of parabolas and hyperbolas into design. He utilized the continuity of hierarchy (Fig. 1-D) in the composition and creation of curved structures (Pantano, 2013).

Fractals are continuous, recurring at different levels to create logical hierarchy in form and structure. In Mandelbrot’s fractal patterns, hierarchical arrangements (Fig. 1-E) in self-organizing are the most important property in that they adapt, harmonize, and evolve (Mandelbrot, 1983, 189). In this view, traditional Persian architecture

hierarchy is at the highest degree of perfection, in which architectural elements create a purposeful harmonizing in structural connections. (Fig. 1-F) shows the hierarchy of connections in Iranian architecture.

#### - Plurality in unity

Leon Battista Alberti, an Italian philosopher, believed that beauty is the conformity of the constituents with the law of nature and its patterns. Unity is inferred in compounds that are self-organized and evolved which represent proportion in perfection (Alberti, 1987). In Alberti's view, the composition is the proportion and conformity of elements in unity. Fig. 1-G shows his principles of composition in the architecture of Santa Maria Novella Basilica. Fractals can be defined as a structure holding a plurality, continuity, and rhythm of the elements that are harmonized and united. Thus, in the Mandelbrot set (Fig. 1-H) unity is the main factor in coordination and self-organizing of hierarchies in fractal patterns. Fig. 1-I shows the organization of the Nasir al-Mulk Mosque ornaments with respect to the Mandelbrot set. The mosque presents the concept of unity in a harmonious combination type of arrays, and therefore, it can be noted that "one of the most important capabilities of architecture is the generation of component unifying metaphors" (Marjouei, Shahedi, Piravi Vanak & Ghasemi Sichani, 2018).

### Experimental foundations of the research

#### • Application of fractal properties in form-based architecture

##### - Main structural elements

The main fractal property is generating self-similar elements and branching in the process of the structural form, whose function in architecture corresponds to the main structural elements (Ramzy, 2015). In this regard, architects of the Gothic period created the structure of a building using a wide range of "rib vault" and "supportive elements". By geometrically emphasizing the use of branches (Figs. 2-A & 2-B) to increase height, they shift the weight of the building onto load-bearing elements and illustrated the walls with painted glasses

narrating religious stories. In this way, "Lightness and colour, together with tall vertical elements give the worshiper an image of the Heaven, an experience of the other world" (Haghani, 2009). Similarly, Art Nouveau architecture implemented another type of branching. For example, the Masia Freixa represents nature-inspired branches in the structure of its columns (Fig. 2-C) that are created to properly transfer structural loads of the building. Although fractal architecture is a relatively new style, the nature-inspired techniques in the original works of Persian architecture illustrate the use of the geometry of natural structures. The correspondence between the architectural elements of the Ali Qapu Palace with the fractal structure (Figs. 2-D & 2-E), self-similarity property is created in the capitals and the hierarchy of connections. Naturalistic branches are considered as internal retaining elements which transfer structural load in this building. As Bovill noted, nature has organized structures and the knowledge of fractal geometry provides a further perspective on the expression of nature's structures in architecture (Bovill, 2000).

##### - Composition of elements and using ornamentation

The hierarchy of connections and geometric progressions are the fundamental factors in organizing the fractal structure. Continuity of connections and ornamentation can be the most significant application in architectural spaces. Architecture is the organization of space and the structure of nature is regarded as an efficient idea in composition, orderly movement of the structural load, and aesthetic concepts in architecture. In this regard, Ramzy (2015) examined the composition and geometric development of ornamentation in the structure of Jerónimos Monastery (Fig. 3-A) with fractal geometry. He pointed out that Gothic architects regarded the idea of order in using nature-inspired geometry as the first law of heaven in creating an ecclesiastical space. Conforming the fractal geometry with hyperbolic space (Fig. 3-B), Crompton noted that hyperbolic space is arising at fractal dimension, and creates a kind of heavenly environment within geometry and structure (Crompton, 2004).

In this point of view, Karbandi is the most salient manifestation in applying geometric progressions



in Persian architecture. “Recent studies on the connections between mathematics, geometry, and Persian art show that Persian artists have made remarkable progress in mathematics and geometry during the Middle Age” (Mohammadian Mansour & Faramarzi, 2012). By examining the geometry of the interior ornaments at the Borujerdi House in Kashan, it can be stated that Persian architects have always

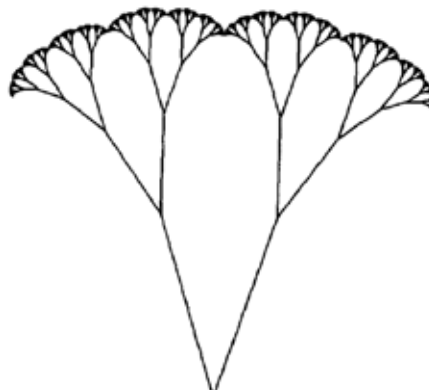
been inspired by nature, and incorporating abstract geometry into a hierarchy of connections. (Fig. 3-C) shows the correspondence of the interior at the Borujerdi House with the characteristics of nature’s geometry in center-oriented geometric expansion.

#### - Structural form

The progress of self-similarity stages, generating rhythm in connections, and integrating a holistic



A) Self-Similar Structure.  
Source: Ruskin, 1857, 124.



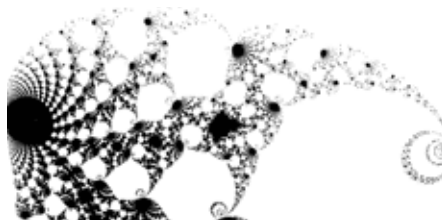
B) Fractal Tree.  
Source: Mandelbrot, 1983, 155.



C) Yazdi-Bandi Structure.  
Source: Raieszadeh & Mofid, 2014, 45.



D) Hierarchy of Connections in Gaudi’s Architecture.  
Source: <https://sagradafamilia.org>



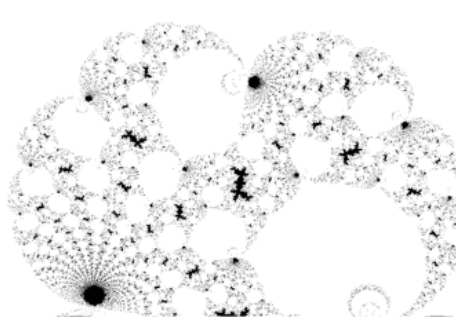
E) Hierarchy in Fractal Patterns.  
Source: Mandelbrot, 1983, 189.



F) Hierarchy of Connections in Persian Architecture.  
Source: Raieszadeh & Mofid, 2014, 225



G) Principles of Alberti Architectural Composition. Source: [www.archdaily.com](http://www.archdaily.com)



H) Unity in the Mandelbrot Set.  
Source: Mandelbrot, 1983, 190.



I) Organization in the Nasir al-Mulk Mosque Ornaments. Source: [www.kindiran.com](http://www.kindiran.com)

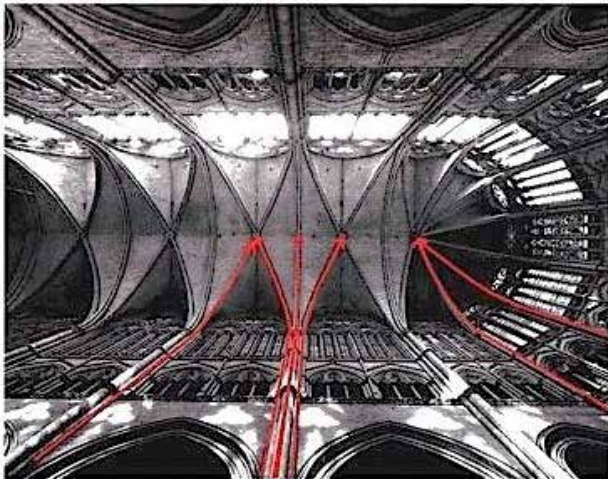
Fig. 1. Properties of Fractal Structure and Form.

pattern into structure and form can be a fractal process in architecture. In this arrangement, the elements in the creation of rhythm and dynamics have a logical relation to the original form, in a manner, the structure has fractal functions in nature. (Jenks, 2002). In this regard, Sala conforms Gothic architecture to the fractal sample (Fig. 4-A). The recurring self-similar forms in the towers, arches, and windows on the church shows the fractal process in the structure form of Gothic architecture, particularly where it is combined with rhythm and function of support elements (Sala, 2006).

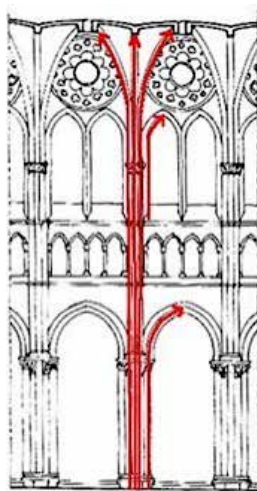
Gothic buildings have created a fractal perspective in their form and structure, with details generating through diversity and multiplicity in a self-

similar geometry. The comprehensive pattern is incorporated into internal elements and the external structural form, and more rhythms are derived from these pattern extending into the windows (Joye, 2007, 177).

In the architectural elements and ornaments of Lincoln Cathedral and Cologne Cathedral (Fig. 4-B), the pointed arches have been expanded in proportions to be applied to the upper windows and in the hierarchy of connections in the exterior structure. In this examination, Lorenz states the fractal concepts used in the composition of elements in the structure of the church. The branching-based system of vertical geometric expansion, whether in the form of “Y-shaped” structures or “rib vault”, dominates the



A) Use of Branches, Chartres Church. Source: Haghani, 2009: 30.



B) Amian Church. Source: Haghani, 2009, 30.

Fig. 2. Main Structural Elements.



C) Nature-inspired Branches in the Structure of Columns, Masia Freixa. Source: www.catalunya.com



D) Architectural Elements of the Ali Qapu Palace in Accordance with Fractal Structure. Source: Authors.



E) Fractal Structure. Source: Ramzy, 2015.



exterior and interior elements at any level in columns, vaults, or towers in the building (Haghani, 2009).

In Art Nouveau architecture (Fig. 4-C), the replication of the towers and architectural elements show several stages of self-similarity in the Montferri Church structure. The use of similar parabolas and hyperbolas not only created a proper rhythm in the form of this church, but also have a function of conveying structural loads like the “supportive element” of the Gothic style.

### Research method

This comparative-descriptive research has an approach to the explanation of the fractal geometry rules in the process of form-based structures in architecture. By examining the sample structure and applications of linear and nonlinear geometry, comprehensive laws of the fundamental fractal structure are studied through adaptation and inference. To demonstrate

the logical connection between the concepts, the correlation method is employed to examine the sample. To achieve the intended results, case studies are considered in evaluating the quality of fractal architecture in this research. The study population includes selected buildings from the Gothic, Art Nouveau, and traditional Persian architecture periods, which incorporated natural geometry as an efficient idea in the structure, composition of architectural elements, and aesthetical concepts of ornamentation. These buildings were selected in a way that provide the quality and diversity required to generalize the results into architecture.

### Research concepts

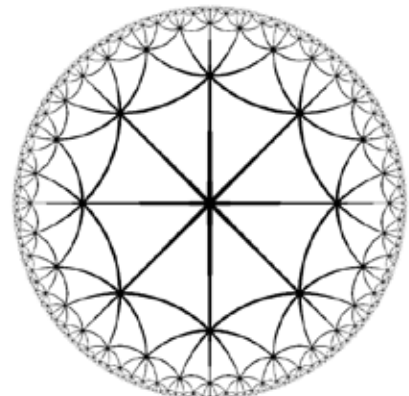
In this study, fractal properties are used as a basis for studying and a description of relevant concepts. Once these properties are converted into concepts and indices (Table 1) the correlation method



A) Geometric Structure of Jerónimos Monastery in Conformity with Fractal Geometry. Source: Ramzy, 2015.



B) Hyperbolic Space Geometry. Source: [https://en.wikipedia.org/wiki/Order-8\\_triangular\\_tiling](https://en.wikipedia.org/wiki/Order-8_triangular_tiling).



C) Internal Ornaments of the Borujerdi House Correspondents in Center-oriented Geometric Expansion. Source: Authors.

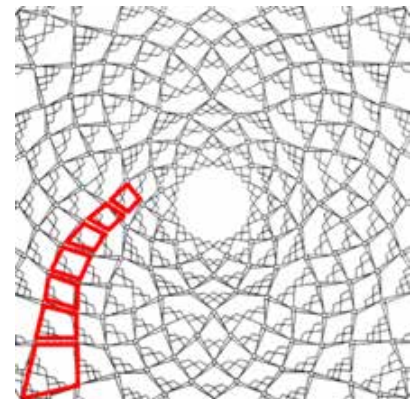


Fig. 3. Elements Composition and Application of Ornamentation.



can explain the logical process in creating form-based structures through applications of linear and nonlinear geometry.

### Research sample

A wide range of natural phenomena like clouds, mountains, rivers, evergreen trees and, ... create similar structures and fractal forms that are the most important indicators of their similarity, continuity, rhythm, and harmony. The sample structure studied was adopted from the “evergreen tree fractal” to achieve the quality required to enable generalized results from the case studies. Through a study of different longitudinal and transverse sections of the sample, seven geometric stages of fractal structures are adopted and applications of linear and nonlinear geometry in the structure of the sample are explained (Table 2).

### Research findings

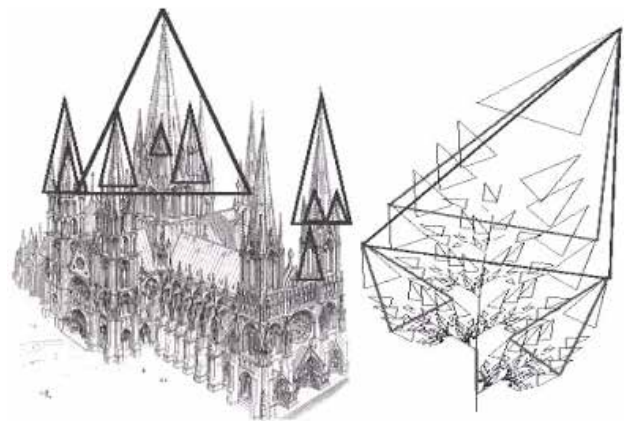
Based on the study of the selected sample (Table 2) it can be stated that fractals follow mathematical and geometric laws to organize structures through hierarchy and geometric expansion. Correlation method defines seven stages including, rotation, transformation, conjunction, branching, the hierarchy of connections, geometric progressions, and unity in explaining form-based structures in fractals. The fractal formation is mainly associated with three laws of self-similarity, self-organization, and unity in structure. The fractal rules explain the process of form-based architecture which is presented in the research model<sup>8</sup> and is used for evaluating the fractal quality of selected buildings (Fig. 5).

### Discussion

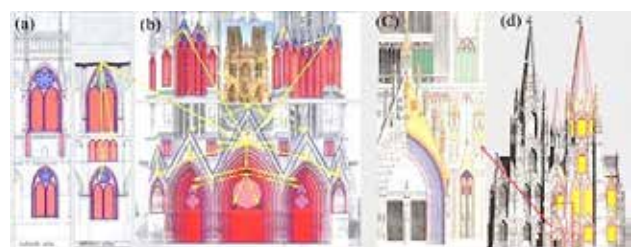
To validate the application of fractal rules in architecture, the Church of Saint Barbara, Basílica de la Sagrada Família, and the Nasir al-Mulk Mosque are being studied. The laws of fractal geometry provide a scientific-applied method in evaluating the selected buildings.

#### • Law of self-similarity

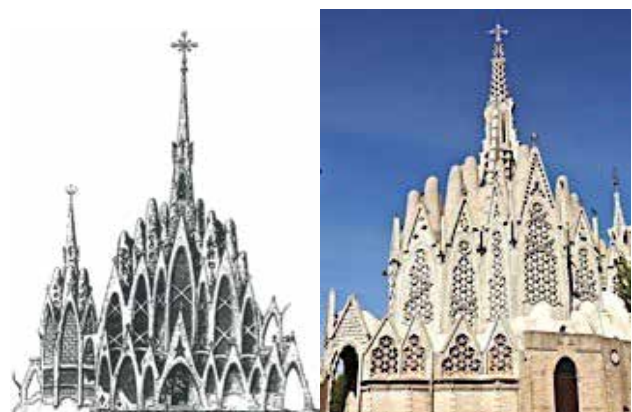
The law of self-similarity in fractal creates a dynamic “rotation” with “transformation” in the geometrical



A) Gothic Architecture in Conformity with Fractal. Source: Sala, 2006.



B) The Ornaments of Lincoln Cathedral (a & b) and Cologne Cathedral (c & d). Source: Haghani, 2009, 140.



C) Recurring Towers and Architectural Elements in Montferri Church. Source: Flores, 1999 & <https://commons.wikimedia.org>

Fig. 4. Structural Form.

nature of the main elements, and the structure is strengthened as it changes in dimensions and form. Subject to the mathematical laws of nature's geometry, such interactions create “conjunction” and “branches” to generate self-similar structure. According to the law of self-similarity, a fractal produces self-similar structures in different levels to establish a hierarchy of connections in the next stage. Fig. 6 shows the process of stable columns formation in accordance with the law of self-

similarity in Saint Barbara Church, Sagrada Família Basílica, and the Nasir al-Mulk Mosque.

In Saint Barbara Church (Fig. 6-A) polygonal columns are intended in which in the next stage, form capitals according to the hierarchy of connections. Abstract nature's geometry of "flower of life" and the golden proportions in this pattern have created crossed and branched arches in the structure of the capitals.

In Sagrada Família Basílica (Fig. 6-B) columns were formed by the process of converting sections from polygons to stars, leading to a circular shape at the top. The diameter and the conversion process of the sections are taken into account by the amount of load on the columns. A connection then forms elliptical capitals where columns divide into branches, as

the branches are arranged to create a hierarchy at different levels. In this loading technique, Gaudí is a pioneer in using natural forms. Yarnall (2010) illustrated the main elements of Gaudí's architecture and conformed them to the structures of nature. Given this match, we can state that Gaudí modified load transfer systems in "supportive elements" of the Gothic period into "branched capitals" as internal retaining elements in Sagrada Família Basílica and performed a rich combination of fractal structures. The Nasir al-Mulk Mosque (Fig. 6-C) features nature-inspired rotation columns. During the Zand period, architects would design rotated columns because of its stability in transferring forces. In addition to giving beauty to the structure, these columns have greater internal elastic property

Table 1. Fractal Properties and the Process of Converting Concepts into Indices. Source: Authors.

Fractal Properties	Concepts	Indicators	Correlation Method
Recurring Elements	Self-Similarity	Curvature - Interchange - Similarity	Linear Geometry Non-Linear Geometry
Hierarchy of Connections	Self-Organizing	Continuity - Consistency – Rhythm	
Plurality in Unity	Unity	Principality – Harmony – Oneness	

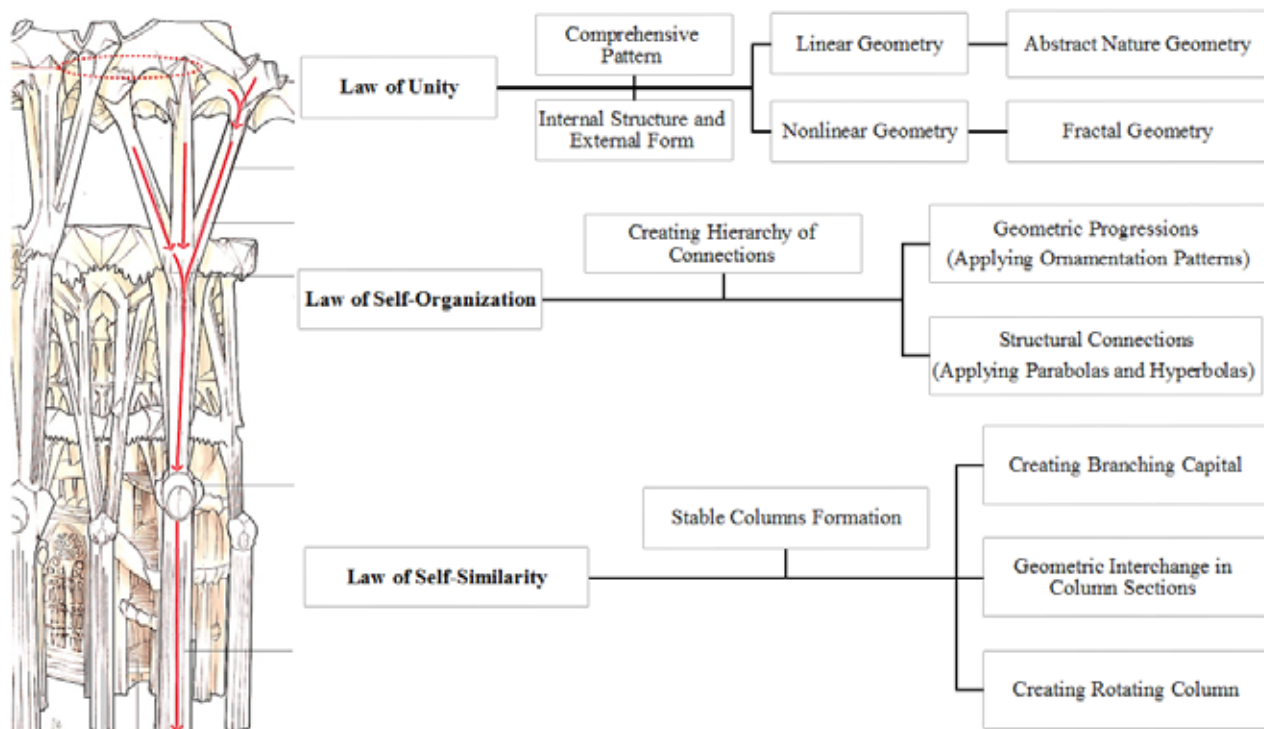
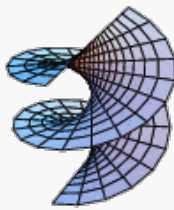

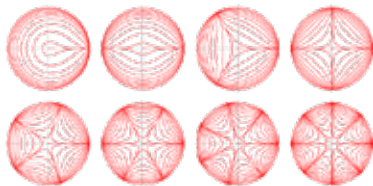

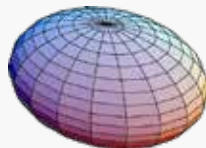

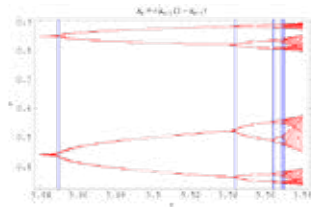

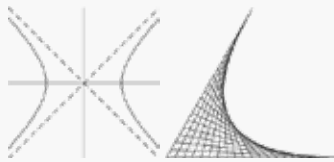







Fig. 5. Research Model, Criteria, and Aspects of Applying Fractal Laws into the Architectural Form-based Structure. Source: Camañas, Márquez & Malet, 2010 & Diagram: Authors.

Table 2. Study of a Selected Sample of Research. Source: Authors.

A Seven-Stage Process of Fractal Structure and Form				
Concepts	Indicators	Correlation Method in Explaining the Application of Linear and Nonlinear Geometry in Sample Structure		Fractal Sample Structure
Self-Similarity	Curvature Interchange Similarity	1. Rotation <sup>1</sup> Source: <a href="https://mathworld.wolfram.com/Helicoid.html">https://mathworld.wolfram.com/Helicoid.html</a>		
		2. Transformation <sup>2</sup> Source: <a href="http://mathworld.wolfram.com/Superellipse.html">http://mathworld.wolfram.com/Superellipse.html</a>		
		3. Conjunction <sup>3</sup> Source: <a href="http://mathworld.wolfram.com/Ellipsoid.html">http://mathworld.wolfram.com/Ellipsoid.html</a>		
		4. Branching <sup>4</sup> Source: <a href="http://mathworld.wolfram.com/Bifurcation.html">http://mathworld.wolfram.com/Bifurcation.html</a>		
Self-Organizing	Continuity Consistency Rhythm	5. Hierarchy of Connections <sup>5</sup> Source: <a href="http://mathworld.wolfram.com/Parabola.html">http://mathworld.wolfram.com/Parabola.html</a> – <a href="http://mathworld.wolfram.com/Hyperbola.html">Hyperbola.html</a>		
		6. Geometric Progressions <sup>6</sup> Source: <a href="http://mathworld.wolfram.com/Fractal.html">http://mathworld.wolfram.com/Fractal.html</a>		
Plurality in Unity	Principality Harmony Oneness	7. Unity and Plurality <sup>7</sup> Source: <a href="http://mathworld.wolfram.com/Rose.html">http://mathworld.wolfram.com/Rose.html</a>		



inspired by natural forms. The connections are then formed by creating polygonal capitals on the main columns where they divide into a number of branches. Branches are arranged to establish a hierarchy of connections in arches.

#### • Law of self-organization

The law of self-organization in fractals creates the “hierarchy of connections” through nonlinear geometry properties of parabolas and hyperbolas, and the structure evolves and details into “geometric progressions”. In the law of self-organization, continuity, consistency, and rhythm are important characteristics in the hierarchy of connections created in the fractal structure. (Fig. 7) shows how the hierarchy of connections and geometric expansions of the ornamentation that were implemented according to self-organization law in Saint Barbara Church, Sagrada Família Basilica, and the Nasir al-Mulk Mosque.

Geometric expansion in Saint Barbara Church (Fig. 7-A) is created in the capitals where an abstract geometry of “flower of life” is expanded into the arches to form hierarchal connections. In this building, the arrangement of connections and the creation of details in accordance with nature’s geometry pattern is extended through ornamentation to express aesthetical and ontological concepts.

In Sagrada Família Basilica (Fig. 7-B) the geometry of parabolas and hyperbolas applied between “branching capitals” and arches produced structural continuities which not only created hierarchical connections, but also have the function of directing light and air into the building. Subsequently, geometric progressions provided various patterns of ornamentation in the interior structure, which are detailed with fractal properties that enhance the aesthetic quality of the spaces.

In the Nasir al-Mulk Mosque (Fig. 7-C) branches have been created in the main capitals, organizing a hierarchy based on the order of connections. The use of fractal geometric expansion in a variety of Persian art ornaments represent the fractal organization in the interior structure of this mosque.

#### • Law of unity

The law of unity composes the main elements of a fractal structure into a comprehensive pattern and



A) The Church of Saint Barbara. Source: [www.khfamost.cz](http://www.khfamost.cz)



B) Basílica de la Sagrada Família. Source: <https://sagradafamilia.org>.



C) The Nasir al-Mulk Mosque. Source: [www.kindiran.com](http://www.kindiran.com)

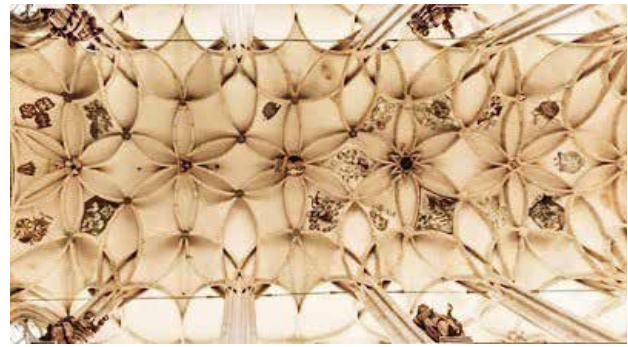
Fig. 6. Stable Columns Formation.

arranges hierarchal of connections in a way that different structural elements are unified. According to the law of unity, fractal structures exhibit unity moving forward and re-originating. The comprehensive pattern is derived from mathematical proportions of the natural geometry that has its root in the Latin word «pattern», which originated from the word «father» and indicates the oneness. In creating structure, nature follows a comprehensive geometry pattern to maintain its continuity with the overall structure, thereby defining one identity (Noghrekar, 2013). Fig. 8 shows how plurality in unity is represented in the structural form of Saint Barbara Church, Sagrada Família Basilica, and Nasir al-Mulk Mosque.

In Saint Barbara Church (Fig. 8-A) abstract geometry of nature has been incorporated into the capitals and developed in the internal structure and external form. Buildings in Gothic architecture have dual expressions: expression of abstract nature's pattern and expression of form. In the composition of interior elements, Saint Barbara Church represents an expression of abstract geometry of nature while it also expresses an exterior form in a combination of a comprehensive pattern. The particular engineering system used in this building is "supportive elements" designed into the exterior of the building. These elements provided the ability to construct a tall structure and use more light and ornamentation into the main body of the building.

In Sagrada Família Basilica (Fig 8-B) the use of "branching capitals" has harmonized the main elements with the associated elements in different levels of hierarchy. In this church, the internal constituent elements and the external structural forms, in harmony with the comprehensive fractal patterns, represent a higher level of self-organization in terms of form and structure compared to the Gothic period. The engineering system of the building, with rotated columns and branched capitals in a form of unique loading capability as internal retaining elements, enabled the construction of a tall structure.

In the Nasir al-Mulk Mosque (Fig 8-C) the concept of unity has created a combination of ornaments in the hierarchy by applying comprehensive patterns



A) The Church of Saint Barbara. Source: [www.khfarnost.cz](http://www.khfarnost.cz)



B) Basílica de la Sagrada Família. Source: <https://sagradafamilia.org>.



C) The Nasir al-Mulk Mosque. Source: [www.kindiran.com](http://www.kindiran.com)

Fig. 7. Hierarchy of Connections and Geometric Progressions.

of ornamentation. The unity of plurality in the architecture of this mosque in the hierarchy of connections originates from oneness. It provides a magnificent portrayal of the diversity and unity of the elements in the internal structure of the mosque. The engineering system contains nature-inspired rotating columns intended to make a stable structure, and branched capitals that enable the distribution of structural load and counteract the thrust of the arches.

## Conclusion

The process of using fractal geometry in architecture is mainly accompanied by the application of the laws of self-similarity, self-organization, and unity, which



create fractal quality in architecture. With respect to the first question about how fractal geometry can be applied in the structural form of architecture, this study provides an insight into using linear and nonlinear geometry and explains the geometry-oriented organization of form-based structures in architecture in seven stages: 1. Rotation, 2. Transformation, 3. Conjunction, 4. Branching in the law of self-similarity, 5. Hierarchy of Connections, 6. Geometric Progressions in the law of self-organizing and 7. Unity and diversity in the law of unity. To answer the second question, this research expresses that fractal geometry function in elements composition, the hierarchy of connections, and ornamentation, follow fractal geometry laws. The law of self-similarity establishes the structural elements of architecture, and the law of self-organization arranges the hierarchy of connections, while the law of unity integrates the structure of the architectural form. In addition, form-based fractal architecture establishes a precise relationship between the composition of elements, structural requirements, and aesthetic concepts of ornamentation through the dual expression of structure and form, to provide internal correlation and external unity between constituent elements and architectural forms.

Considering (See Table 2) and the research model (See Fig. 5), it can be stated that in fractal architecture seven-stage of applying fractal geometry in column formation, elements composition, the hierarchy of connections, and geometric expansions, are evaluated in terms of fractal characteristics in structure and form (Table 3). In addition, this model describes principles in presenting nature-inspired engineering systems that provide a scientific-applied method for enhancing the quality of architecture.

## Endnotes

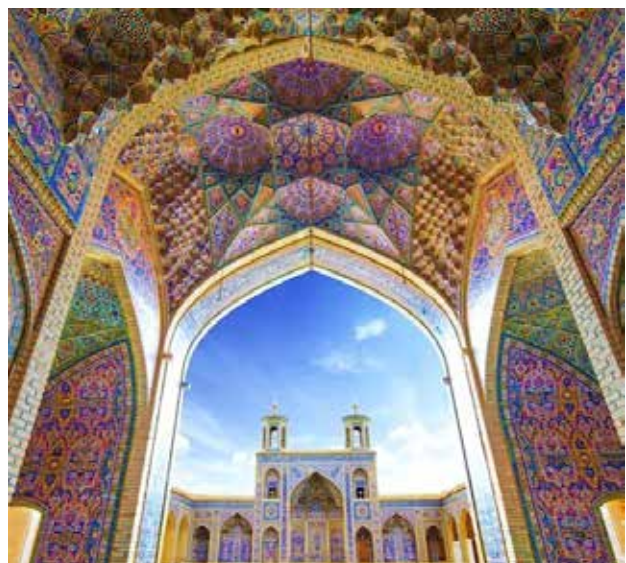
1. helicoid: a curve that rotates between two points.
2. super ellipse: ellipses with a Cartesian equation.
3. ellipsoid: creates a quadratic ellipse in Cartesian coordinates.
4. bifurcation: performs a periodic segmentation.
5. parabola and hyperbola: variety of curves by intersecting with the cone.
6. geometric progressions: displays geometric Patterns.
7. rose progression: the circles are six times symmetrical, implying a pattern of circles in plurality and unity.
8. fractal-based architecture model (FBAM): Form-based architecture research model, cited in this article.



A) The Church of Saint Barbara. Source: [www.khfarnost.cz](http://www.khfarnost.cz)



B) Basílica de la Sagrada Família. Source: <https://sagradafamilia.org>.



C) The Nasir al-Mulk Mosque. Source: [www.kindiran.com](http://www.kindiran.com)

Fig. 8. Plurality in Unity in Structural Form.



Table 3. Evaluating Fractal Qualities of the Selected Buildings. Source: Authors.

Fractal Geometry Laws	Architectural Elements	Fractal Form-Based Model	Linear geometry: Abstract geometry of nature. Nonlinear geometry: The geometry of natural structures.	Saint Barbara Church	Basilica Sagrada Familia	The Nasir al-Mulk Mosque
Self-Similarity	The columns Formation	1. Rotation	Creating a polygonal or rotating columns.	✓	✓	✓
		2. Transformation	Geometric interchanges in columns sections.	✓	✓	✓
		3. Conjunction	Connecting by creating round, elliptical, and polygon sections.	✓	✓	✓
		4. Branching	Creating branching and naturalistic capitals.	✓	✓	✓
Self-Organization	Elements Composition and Ornamentation	5. Hierarchy of Connections	Applying parabolas and hyperbolas in creating structural connections.	✓	✓	✓
		6. Geometric Progressions	Geometric progressions using ornamentation.	✓	✓	✓
Unity	Structural Form	7. Unity and Plurality	Composition of comprehensive pattern and creating internal connections and external unification.	✓	✓	✓

## Reference list

- Alberti, L. B. (1987). *The Ten Books of Architecture*. New York, US: Dover.
- Ashrafi, N. (2017). Recognition of Islamic Urbanism and Architecture with Chaos Theory and Fuzzy Rule. *Bagh-e Nazar*, 13(43), 79-94.
- *Basilica of Santa Maria Novella*. Retrieved May 8, 2020, from: <https://www.archdaily.com>.
- *Bifurcation*. Retrieved May 8, 2020, from: <http://mathworld.wolfram.com/Bifurcation.html>.
- Bovill, C. (2000). Fractal Geometry as Design Aid. *Journal for Geometry and Graphics*, 4(1), 71-78.
- Camañas, R. & Márquez, R. & Malet, O. (2010). *Un Símbolo De Barcelona*. Barcelona, Spain: La Vanguardia.
- Crompton, A. (2004). *The Fractal Nature of Everyday Space*. University of Manchester, UK: School of Environment and Development.
- *Ellipsoid*. Retrieved May 8, 2020, from: <http://mathworld.wolfram.com/Ellipsoid.html>.
- Flores, C. (1999). *Josep María Jujol*. Madrid, Spain: Revista Arquitectura, COAM.
- *Fractal*. Retrieved May 8, 2020, from: <http://mathworld.wolfram.com/Fractal.html>.
- Haghani, T. (2009). *Fractal Geometry, Complexity, and The Nature of Urban Morphological Evolution*. Ph.D. Dissertation. School of Architecture, Birmingham City University, UK.
- *Helicoid*. Retrieved May 8, 2020, from: <https://mathworld.wolfram.com/Helicoid.html>.
- Jencks, C. (1997). *The Architecture of Jumping Universe*. Sussex, UK: Academy Editions, Wiley.
- Jencks, C. (2002). *The New Paradigm in Architecture: The language of postmodernism*. Connecticut, US: Yale University Press.
- Jencks, C. (2011). *The Story of Post-Modernism: Five Decades of the Ironic, Iconic and Critical in Architecture*. Sussex, UK: Wiley.
- Joye, Y. (2007). *A Tentative Argument for The Inclusion of Nature-Based Forms in Architecture*. Ph.D. Dissertation. Faculty of Arts and Philosophy, Universiteit Gent, Belgium.
- Mandelbrot, B. (1983). *The Fractal Geometry of Nature*. New York, US: W.H. Freeman.
- Marjoui, A. & Shahedi, B. & Piravi Vanak, M. & Ghasemi Sichani, M. (2018). Architecture as Extensive Mind. *Bagh-e Nazar*, 15(64), 33-46.
- *Masia-Freixa*. Retrieved May 8, 2020, from: <http://www.catalunya.com/masia-freixa>.
- Mohammadian Mansour, S. & Faramarzi, S. (2012). Comparison of King Knot Pseudo-Periodic Order with Silicon-Like Crystalline Structure. *Honar-ha-ye Ziba (Honar-ha-ye Tajassomi)*, 50, 69-80.
- *Montferri Church*. Retrieved May 8, 2020, from: [https://commons.wikimedia.org/wiki/Category:Mare\\_de\\_Deu\\_de\\_Montserrat\\_de\\_Montferri](https://commons.wikimedia.org/wiki/Category:Mare_de_Deu_de_Montserrat_de_Montferri).
- Moussavi, F. (2009). *The Function of Form*. Harvard Graduate School of Design: ACTAR.
- *Nasir-al-Mulk Mosque*. Retrieved May 8, 2020, from: <http://www.kindiran.com/en/Gallery/The-Nasir-al-Mulk-Mosque>.
- Nogherekar, A. H. (2013). *Man's Relationship with Nature*

and Architecture. Tehran, Iran: University of Science and Technology.

- *Order-8 triangular tiling*. Retrieved May 8, 2020, from: [https://en.wikipedia.org/wiki/Order-8\\_triangular\\_tiling](https://en.wikipedia.org/wiki/Order-8_triangular_tiling).
- Pantano, M. (2013). *Reading Gaudi's Great Book of Nature: Reconsidering the Peripheral Reception of Proto-Environmental Architecture*. US: University of Pennsylvania.
- *Parabola- Hyperbola*. Retrieved May 8, 2020, from: <http://mathworld.wolfram.com/Parabola.html-Hyperbola.html>.
- Rahmani, E. & Etessam, I. & Mokhtabad, M. (2017). An Analysis of How to Criticize and Evaluate Avant-Garde Architectural works. *Bagh-e Nazar*, 14(50), 5-22.
- Raiszadeh, M. & Mofid, H. (2014). *Reviving the Lost Arts: Foundations of Traditional Architecture in Iran According to Hossein Lorzadeh*. Tehran, Iran: Mola Publishing.
- Ramzy, N. S. (2015). *The Dual Language of Geometry in Gothic Architecture: The Symbolic Message of Euclidian Geometry Versus the Visual Dialogue of Fractal Geometry*.

Egypt: Sinai University.

- *Rose*. Retrieved May 8, 2020, from: <http://mathworld.wolfram.com/Rose.html>.
- Ruskin, J. (1857). *The Elements of Drawing*. London, UK: Spottiswoode & Co.
- *Sagrada Familia Basilica*. Retrieved May 8, 2020, from: <https://sagradafamilia.org/en/photo-gallery>.
- Sala, N. (2006). *Fractal Geometry and Architecture: Some Interesting Connections*. Mendrisio, Switzerland: Università Della Svizzera Italiana.
- *Santa Barbara Cathedral*. Available from: <http://www.khfarnost.cz/st-barbaras-cathedral>.
- *Superellipse*. Retrieved May 8, 2020, from: <http://mathworld.wolfram.com/Superellipse.html>.
- Taghvaei, V. (2012). From Order to Disorder in Nature and Architecture. *Hoviatshahr*, 6(11), 39-52.
- Yarnall, K. M. (2010). *Barcelona's Natural Wonder*. USA: National Geographic Society.

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