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## Explanation of Geometrical System of zone of transition in Gonbad-e Sorkh in Comparison with Common Pattern of Seljuk's Zone of Transition\*

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### Abstract

Changing the square into the circle in the structure of dome chambers was a main problem in Iranian architecture. The solution to this problem has led to the formation and creation of capable methods and techniques in the structure of a part of dome chambers that changes square chamber below into the round dome above it. This phenomenon has been considered as “the zone of transition” (transitional zone) in many studies on Iranian architectural history, but it has been less seriously studied in a strict sense as an important independent part. On the other hand, the way square chamber corners corbel has made different forms of structures that is called “squinch”. The four squinches together with the middle parts between them organize the mentioned zone (zone of transition). In some studies, the zone of transition has been described considering the load-bearing elements of corbelled form of squinches. Some experts have indicated different forms of squinches such as “stalactite squinch”, “conical squinch”, “trie-lobed squinch”, “squinch net” and some others have talked about it considering materials, ornaments and constructional styles. Although these considerations present some aspects of transitional zone such as historic, structure and form, it rarely has been explained as a complete evolution and clear classification. Geometry, as the essence and nature of this process (changing square into the circle), is certainly a decisive item. Therefore, this research is going to explain the zone of transition from the view point of geometry, considering the geometrical system between square and circle that leads to a regular polygon as the base of dome. According to this geometrical approach, the article first reviews two old methods used before Seljuk era and then, explains the elegant form of Seljuk zones of transition originated in Nezamolmolk dome of Isfahan, as the most fashionable pattern existing in Iran for more than three centuries. In the next phase, which is the context of this research, the transitional zone of Gonbad-e sorkh is explained as a novel form leading to a hex decagon. Although the dominant pattern also eventuates a hex decagon, the case of Gonbad-e sorkh, as an authentic Seljukian monument, does not follow it and presents a different solution. The difference between these two solutions is presented by a comparative analogy. First, the process of changing square into octagon and then hexadecagon in Seljukian fashionable pattern is described. Then, it is compared with the process of changing square into hexadecagon in Gonbad-e sorkh. By this comparison, the paper finally presents the zone of transition of Gonbad-e sorkh as a “shortcut pattern” of a novel geometrical arrangement of transitional elements.

### Keywords

Zone of transition, Squinch, Gonbad-e sorkh, Octagonal prism, Geometry.

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## Introduction and problem statement

The process of changing square into the circle in Iranian dome chambers and the how of this process is a problem that has created many architectural forms and techniques in the structure of domes' zone of transition over the centuries of trial and error. One of the highlighted points of this evolutionary trend is the flow started with the formation of zone of transition of Khaje Nezamolmolk dome of Isfahan Jameh mosque in the second half of the 5th century HA. This elegant form of structure and geometry, as one of the most important achievements of the Seljuk era, has been as the dominant pattern of transitional zones in vast areas of Iran's plateau for centuries. The building of Gonbad-e sorkh of Maragheh is one of the most authentic Seljuk monuments; although it has obvious Seljuki characteristics such as brickworks and Kofic inscriptions in its structure of zone of transition, it presents a different solution to the common tradition influenced by the achievement of Nezamolmolk dome. This study is going to explain this new solution in the form of a systematic structural set through the presentation of the formation and placement of regular polygon under dome on the four walls of Gonbad-e Sorkh.

## Theoretical foundations and literature review

As stated before, most researchers who have somehow studied the Gonbad-e sorkh have been influenced by its Seljuki manifestations especially the excellent quality of its brick decorations (Behnam, 1963: 3 and Pop, 1994: 44, and Hatam, 1999: 132). Some experts have even called it the most beautiful brick building in Iran, or in the world (Schroeder, 2008; 1231). In other words, the variety of bricklaying techniques in the decorative forms, knots and Kofic inscriptions is manifested in this building in such a way that implicitly affects other aspects of its architecture. Even the brilliant combination of tile and brick on

its entrance that is identified as the first instance of such a presence of tile by several researchers (Behnam, 1963: 4) has been also affected by the reputation of this building as a brick masterpiece. The zone of transition of this building has not received great attention as it should. Andre Godard says: "... it is a quadrangular building with a dome-shaped roof placed on stalactites that are still intact" (Godard, 2008: 286). As presented in the quotation, along with a detailed presentation of its decorative properties, brickwork, and inscriptions, he ignores its zone of transition by calling it stalactite. The same is seen in some other studies, i.e. ignorance of its zone of transition. One of the other studies on Seljuk architecture has similarly ignored it through the use of the word "stalactite" in expressing the structure of corners and zone of transition (Hatam, 1999: 133). Meanwhile, Gholam Hossein Memarian evaluated the zone of transition of Gonbad-e sorkh with a closer look and gives a more obvious picture of its structure (Memarian, 2015: 407) through his proposed method called structure-shape (ibid. 392).

Moreover, by looking at the field of upstream resources, we can identify many studies on the zone of transition that have always focused on corner corbel of the room, which bears the weight of the suspended part of the dome. Thus, the term "Squinch" that is defined as an arch, a structure, or a corbelling set that diagonally makes the angles of a square-shaped room tangent under the dome above it (Smith, 2001:129 and M. Harris, 1984: 466 and S.Myers, 1970: 224 and Edwards, 1999: 72) is used as the main keyword of these studies more than any other term. Phrases such as Muqarnas<sup>1</sup>, se-Avizei squinch<sup>2</sup>, badbadaki squinch<sup>3</sup>, Makhruqi squinch<sup>4</sup>, etc more or less illustrate how this structure is. Simialrly, in the Persian literature, the most widely used expression for this concept is "Gushe sazi" instead of "squinch" that was expressed by the deceased Pirnia and his followers<sup>5</sup> (Pirnia, 1991: 16, and

Memarian, 2015: 385, and Bozorgmehri, 2013: 166) and in some cases, the term “Gushvar” has been used with the same meaning in the quotations from old masons of Khorasan (Zamani, 1971: 2). The types of squinches can also be found in the references with concepts as “Eskondj”, “Filpoosh”, “Patgin” so on (Pirnia, 1991: 27, and Memarian, 2015: 400).

In a more comprehensive view, what is known as the zone of transition<sup>6</sup> in the studies of this area is a set of intermediate forms and elements between the square (room) and the circle (dome), which will certainly include the squinches. In general, studies on the zone of transition, as it was seen of mentioned expressions, usually evaluates the diversity of architectural forms in a formal aspect while some of them are also have considered this subject from view point of structure (Minston, 1973: 135). Meanwhile, a geometric approach can also be found in accordance with the nature of the subject in a supplement to Jamshid Kashani’s translation of the “Thesis on the arch and the sun” (Jazbi, 1987: 91).

## Methods

Since the process in zone of transition (changing of square into the circle) more than anything reveals the geometric nature of this structure, this study also employed a geometric approach in its analysis of this process. Accordingly, the study explains the manner of arrangement of load bearing geometric forms from the top of the corners of square room to a regular polygon as the base of the dome in the form of a “geometric system leading to a circle.” The paper, firstly, describes the structure of the common Seljukid transition zones while referring to previous experiences and achievements from the view point of the mentioned geometric system, and then evaluates the zone of transition of Gonbad-e sorkh from the same point of view in a comparative analogy with comparable and equivalent samples using a historic analytical method. Therefore, presentation of visual documentations obtained from field studies will be decisive in this analogy and inference.

## Findings

### • Formation of two main patterns, of transition zones before Seljuk era

As stated in the literature review, most researchers have explained the shapes and architectural features in relation to transition zones. In better words, in expressing and explaining of a geometric process, we mostly face non-geometric approaches influenced by features such as shape, style, structure, or even history. Consequently, instead of precise, transparent, and generalized geometric definitions, there are many of formal, structural, or descriptive interpretations that are not clear and obvious as they are expected to be.

Seyyed Alireza Jazabi is one of the few scholars who has explained geometry referring to the arrangement of the eight, sixteen and 32-sided prisms on each other in passing from square to circle (Jazabi, 1987: 91). In a similar view, the zone of transition can be interpreted as a “geometric system”, expresses the manner of arrangement of load bearing elements in this zone. From this point of view, if we look at the evolution of this phenomenon, we can clearly see two main pattern of this geometric system during the pre-Seljuk era experiences. “Ardeshir Palace” in Firouzabad is one of the oldest instances that although has not created a clear form of polygons, it is important as the first step in the apparent formation of a zone between the lower square room and the upper round dome (Pope, 1976: 104) (Figure 1, a). It is a structure that uses a combination of cone and cylinder in such a way that four half cones and body of a cylinder (stem of the dome) are merged in four corners of the room and placed on the square (Golchin, 2009: 24).

The second step of this process, which should be really considered as a turning point in the formation of a precise geometric system, is the brilliant achievement of the zone of transition in “Dome of Amir Ismail Samani” in Bukhara, where eight identical Ribes alternatively cover the corners of the dome-shaped room, thus providing a zone of transition in the form of a precise and solemn



Fig. 1. Two main patterns of the zone of transition before the Seljuk era.

A (Left): Dome of Amir Ismail Samani in Bukhara

Photo by: Hojjat Golchin. B (Right): Palace of Ardeshir in Firuzabad Fars. Source: Hermann, 1994: 96.

structure of a regular eight-sided prism (Fig. 1b). The significance of this achievement in the evolution of the transition zones is so much that some scholars have called it the beginning and a new point of departure (Hillenbrand, 2008: 289 and Schroeder, 2008: 1159).

● **Seljuk zones of transition: a clear example of the formation of a geometric system leading to a circle**

“The Regular Stratified System of Geometry” is the interpretation of this article to get the next achievement of this evolution, when it is not possible to keep a large dome with a diameter of more than fifteen meters on eight-sided area and a regular octagon plan, where the dome field suspends in the corners. Thus, sixteen smaller arches alternatively cover the angles of lower octagon in the form of a bridge and next to each other, form another floor in the form of a regular sixteen-sided prism on the lower eight-sided prism for the first time in the dome of Khaje Nezamolmolk of Isfahan in the second half of the fifth century AH7 (Fig. 2, A and B).

This elegant form of geometry and structure is the best manifestation of what this study refers to as “a geometric system leading to a circle”. Regardless of some details of squinches, in its general form, it is the dominant geometric system

of the transition zones in the Seljuk and even the Ilkhanid eras (Fig.3).

Although this method is not dominant in the northeast and Transoxiana (Great Khorasan) (Schroeder, 2008: 1217), it has become the most comprehensive method of constructing transition zones in the vast areas of the Iranian plateau for more than three centuries (Godard, 2008: 95 and Schroeder, 2008: 1217 and Wilber, 2014: 68)8. The importance of this achievement is such that even in instances such as the dome of the Jame Mosque of “Qorveh” or the dome of “Imamzadeh Aqil” in Bijar, where the small dimensions of the dome does not necessarily require the formation of a sixteen-sided basis, this solution is still considered as a pure “structural geometry pattern” (Golchin, 2012: 248) (Fig. 4). In addition to other common features of such small domes, especially from the Seljuk period, with the Gonbad-e sorkh, they certainly provide a better possibility of analyzing the structure of transition zones in a comparative analogy due to the similar dimensions of their dome span.

**Discussion**

The geometrical system in transition zone of Gonbad-e sorkh of Maragheh

First of all, it must be acknowledged that the

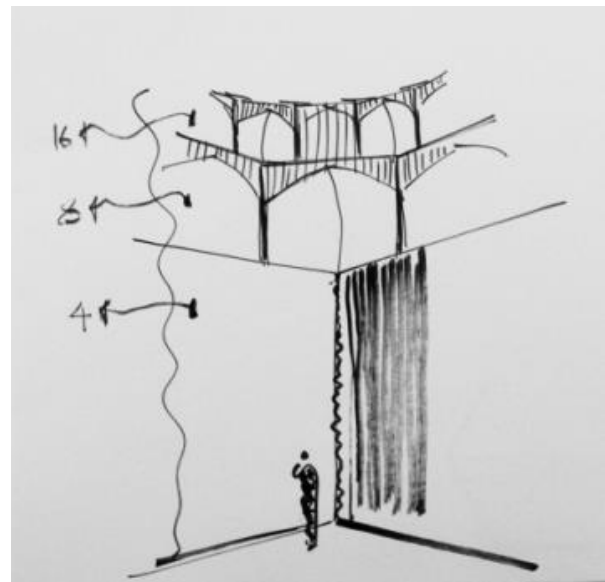


Fig. 2. Up-left: The exterior view of the geometric system of zone of transition of the dome of Nezamolmolk in Isfahan Jami Mosque. Up, Right: The concept of the “regular stratified system of geometry” consisting of eight and sixteen-sided prisms (the architectural achievement of Nezamolmolk dome), Source: drawn by Hojat Golchin. Left image: The the stratified geometric system of transition zone in the external view of the Nezamolmolk dome. Source: Godard, 2008: 47. Fig. 2. Bottom, left: The Geometric concept of transition zone of Nezamolmolk Dome (stratified Eight and Sixteen-sided prisms), a View from inside. Source: Golchin, 2012: 248.

existence of great similarities in the architectural style of the Seljuk, Ghaznavid, Buyeid and Ilkhanid eras leads to speculations and disagreements among these periods in the face of a building without history. Furthermore, there are many buildings that have been repaired or rebuilt during the Seljuk era, or their Seljuk bases have changed in the next period. However, the

existence of the term “Bonia” (was built) at the beginning of the inscription on the entrance of Gonbad-e sorkh, dated to 542 AH indicates the originality and creditability of Gonbad-e sorkh as a Seljuk monument in comparison with samples without history or samples including terms such as “restoration” and “repair”,. The internal map of the dome room is square with



Dome of Tajolmolk of Isfahan- Seljuk



Dome of Jami mosque of Borujerd



Dome of the Jami mosque of Qazvin- Seljuk



Dome of the Golpayegan mosque - Seljuk



Dome of Ardestan mosque Seljuki



Dome of Varamin-Ilkhani Mosque

Fig. 3. Examples of the “Regularly stratified System of geometry” in the zone of transition. Photo by: Hojat Golchin.



Fig.4. Left: The permanency of the Seljuk pattern of transition zones influenced by architectural achievements in nezamolmolk dome, in the small dimensions of dome.

Right: Dome of Jameh Mosque of Qerveh (Photo by Hojat Golchin). Left: Dome of Imamzadeh Aqil in Yasukand Village of Bijar. Photo by: Isa Esfanjari.

a side length of 5.59 meters (Hatam, 1999: 133) and just over the two shaded arches in the inner walls of the dome, the zone of transition can be seen (Fig. 5a). In the exterior, this zone is mounted as an eight-sided prism on the cubic form of the room. Four sides of this eight-sided prism are alternatively bridged along the walls of the dome chamber, and four other sides, have been

spanned diagonally on the corner (Figure 5b). This geometric structure is very evident in the exterior view of a wide range of Iranian domes, especially in Seljuk and Ilkhani eras<sup>9</sup>. In fact, except for a few cases, like Nizamolmolk dome of Isfahan presented the system of stratified prisms in its exterior (Fig. 5a), In most other samples, the external representation of the zone of transition is

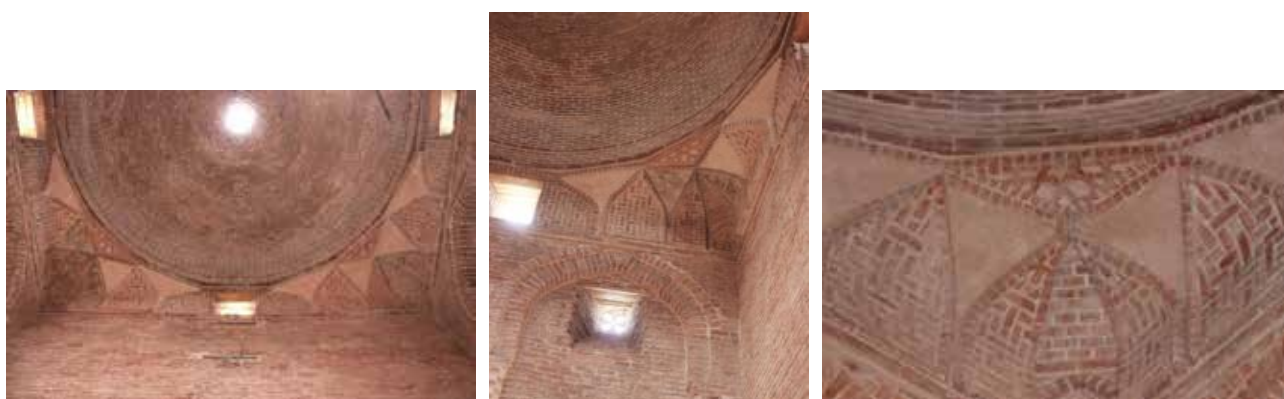


Fig. 5a (Up): The zone of transition of Gonbad-e sorkh and the form of its squinches, a view from inside. Photos by: Hojat Golchin. 5b (Middle) : The exterior of the Gonbad-e sorkh and its zone of transition in the form of an eight-sided prism based on the cube-shaped dome chamber. Photo by Andrade Godard. Source: Godard, 2008: 287.

summarized in the same eight-sided prism (Fig. 6). However, the point which is cleared at first glance at the Gonbad-e sorkh is the short height of the transition zone; although in its outer appearance it presents higher in connection with the outer covering of the dome, its shortness compared with the examples is quite evident (Fig. 5b and 6). This phenomenon is undoubtedly influenced by the manner of arrangement of transition zone elements. One of few studies in this category that has looked closer at the Gonbad-e sorkh of Maragheh, has called its transition zone structure as a fusion of “arch and squinch net” (Memarian, 2015: 407). Apart from these expressions, the demonstration of transition zone in this study can be useful in the explanation of its geometric system. (Fig. 7). From this perspective and through its comparison to common Seljuk zone of transitions with a closer look, in particular in comparison to the two similar buildings of the Qerveh Mosque

and Imamzadeh Aqil in Bijar, which are 5.5 and 6 meters in diameter, it leads to clear important points (Figures 4 and 5-a). First, in both instances, the four main arches are independently seen in the corners of the room and in the octagonal area, while in the Gonbad-e sorkh a set of corbelled niches<sup>10</sup> have created the geometric content of corners as the form of squinches. Another issue is the regular polygon as the base of the dome. Both patterns, ( Gonbad-e sorkh and the other two examples), eventually lead to a regular hexadecagon( sixteen-sided polygon) However, the basic problem is how to formulate this polygon which creates a completely different geometric system in the transition zone of Gonbad-e sorkh. Based on the common pattern of Seljuk transition zone, the obtained hexadecagon in the cases of the Qorveh Mosque and Imamzadeh Aqil is formed in two steps and during a stratified structure. Thus; first the square changes into octagon, and then



Dome of the Jami Mosque of Qazvin (Godard, 2008, 4: 24)



Dome of Borujerd Jami Mosque (Photo by Hojat Golchin)



Dome of Tajolmolk of Isfahan (Godard, 2008, 3: 93)



Dome of Ardestan Jami Mosque (Pop, 2008, 8: 279)



Dome of the Golpayegan Jami Mosque (Pop, 2008, 8: 277)



Dome of Jami Mosque of Varamin (Godard, 2008, 3: 126)

Fig. 6. The dominant exterior view of the transition zone in Seljuk and Ilkhani domes.



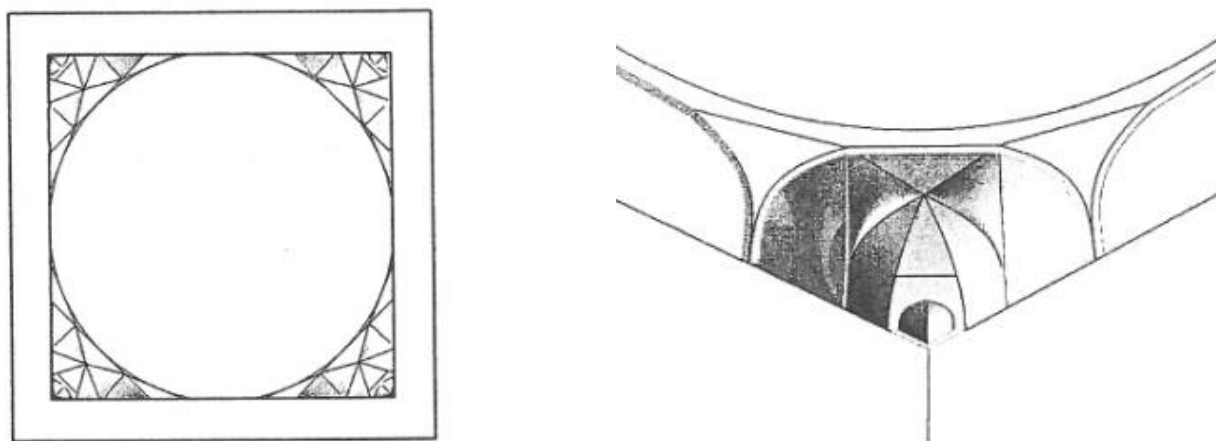


Fig.7. The form of squinch in Gonbad-e sorkh, quoted by Gholam Hossein Memarian. Source: Memarian, 2015: 407.

in the same process, the regular octagon changes into the hexadecagon. However, the mentioned hexadecagon in the building of Gonbad-e sorkh of Maragheh is formed through the precise arrangement of load-bearing elements in one step and is placed on the dome chamber with the least possible height. The implementation of this method undoubtedly indicates its architect's correct understanding of the nature of the regular forms and their embedment in each other. As this way by eliminating one step or in other words, by creating a "shortcut pattern," the possibility of direct access to the hexadecagon is obtained through a new regular geometric system (Fig. 8). However, the structural logic and hierarchy of the form of geometrical system as eight and sixteen

are absent in Gonbad-e sorkh. This makes a clear difference in the internal proportions of the dome with the common model of the Seljuk era, which reminds the shorter nature of the zone of transition more than before (Figure, 9). Moreover, in view point of structural usage it is difficult to implement this method in large spans since the compensation of such a rate of corner corbelling is not possible in this way and at this low altitude. Maybe this is the main reason for the failure of this method in the form of an inclusive model. However, the lack of popularity of this model for any reason highlights the Maragheh Gonbad-e sorkh in terms of providing a new solution and a new achievement of the geometrical system of transition zone.

## Conclusion

Creating a dome on a square base during the historical periods has created a variety of forms of a "geometric system leading to a circle" in the zone of transition of Iranian dome chambers, arranged the establishment of the load bearing elements, from the corner of the top of square room into a regular polygon below the dome. In the Seljuk era, a great architectural achievement regarding the construction of zone of transition was created through the construction of the dome of Khajeh Nizamolmolk in Jame Mosque of Isfahan that realized passage from square to circle in the form of a stratified geometric system consisting of eight and sixteen-sided prisms.

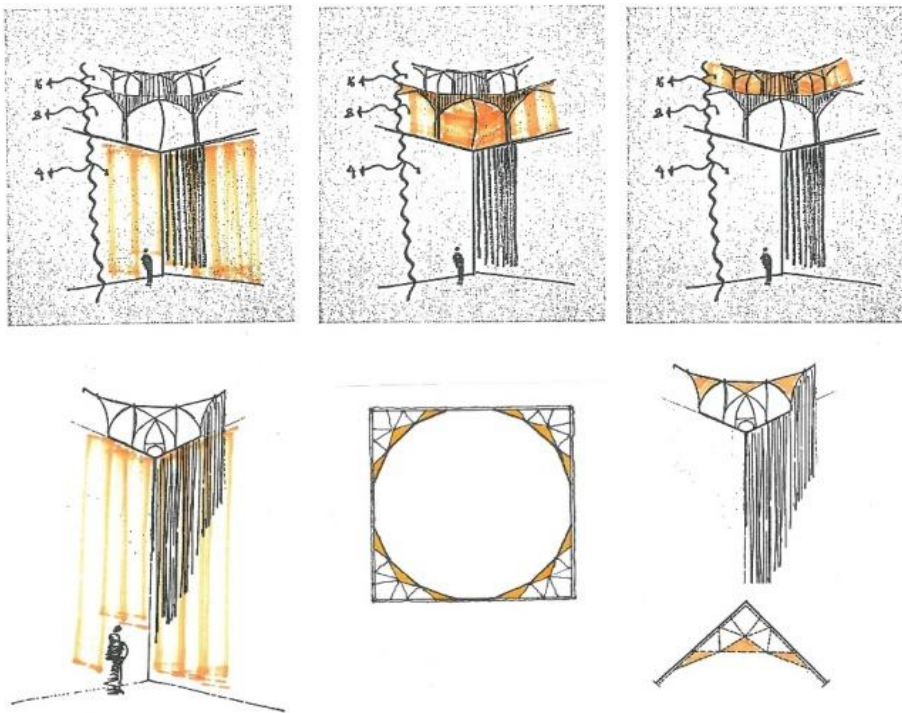
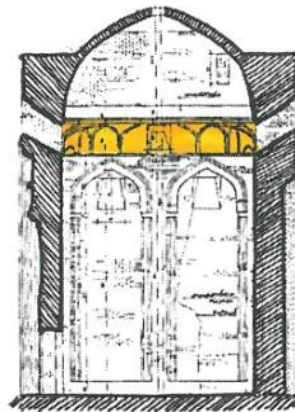


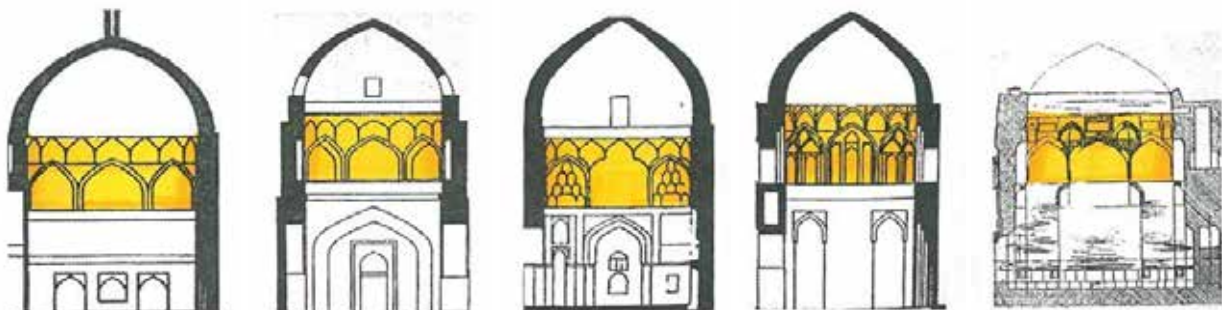
Fig. 8. Geometric analysis of transition zone pattern of Gonbad-e sorkh, in comparison to the dominant pattern of Seljuk zones of transition.

Upper image: The hierarchy of the stratified eight and sixteen-sided prisms in the common geometric system of the Seljuk zones of transition to create a regular hexadecagonal plan. Source: Golchin, 2012: 248.

Lower image: direct changing of square into regular hexadecagon in zone of transition of Gonbad-e sorkh. Source: Drawing by Hojat Golchin.



Gonbad-e sorkh of Maragheh, taken from Athar magazine No. 20



Jameh Mosque of Qazvin (Ganjnameh, 2004)

Jameh Mosque of Qerveh (Ganjnameh, 2004)

Jameh Mosque of Golpayegan (Ganjnameh, 2004)

Jameh Mosque of Zavareh (Ganjnameh, 2004)

Imamzadeh Aqil Bijar (Esfnjari, 2005)

Fig. 9. The apparent difference in the height of the zone of transition in the internal proportions of Gonbad-e sorkh in comparison with the proportions of other domes of the Seljuk dome chambers.

This elegant form of geometry and structure has created the domes on a regular hexadecagon plan in vast areas of Iran's plateau for more than three centuries. While modifying the altitude of its transitional zone, Gonbad-e sorkh of Maragheh, as one of the most significant Seljuk buildings, created a "shortcut pattern," realizing direct access to the regular hexadecagon in the form of a different geometrical system. Although this pattern does not become common due to its inefficiency in large spans, it distinguishes Gonbad-e sorkh of Maragheh in terms of providing a new solution for creating a round dome on a square room.

## Endnote

1. Stalactite squinch (Ibrahim, 1975: 12)
2. Tri-lobed squinch (Edwards, 1999: 75)
3. Kite shaped squinch (Clevenger, 1968: 61); it refers to the shapes such as "Pabarik" and "shaparak" appearing in squinch; all of these shapes together are called "squinch net" in some other sources (O'kane, 1976: 25).
4. Conical squinch (Elkhatib, 2012: 160)
5. "Pirnia" believed that the term "squinch" is taken from the Persian word "Eskondj" (one of the forms of Squinch in Iran). (Pirnia, 1991: 23)
6. Zone of transition. This phrase, along with the word "squinch", is one of the most important keywords of mentioned studies, and in some cases used interchangeably with "Transitional Elements" (Minston, 1973: 135). Pirnia has used the Persian expression of "Chapireh" as an alternative to this phrase (Pirnia, 1991: 8).
7. In "A survey of Persian Art," Khaje NizamoMulk dome of Isfahan is expressed as the first dome sitting on a crown of sixteen arches, dating back to about 473 AH. Schroeder, 2008: 1219).
8. the most important area of these zones is the area known as "Jebal state" in historical resources with the centrality of Isfahan (Estakhary, 1968: 164). Maghdasi called this geographical area "kuhestan" (the land of mountains) and included Isfahan, Ray, Hamedan, Kashan, Qom, Qazvin and Zanjan (Maghdasi, 1361: 572), and elsewhere, Qom and Karaj are the sub regions of Isfahan (ibid: 587). And in "Nozhat al-Qolub" this area is called "Araq-e Ajam" and refers to Isfahan in its context, including the cities that Maghdasi has mentioned (Mostofi, 983: 47).
9. Andre Godard has documented the text of the frieze and in his book, "Asar-e Iran", has written: "this tomb was made in eleventh of Shavval in the year of 542 AH" (Godard, 2008: 94).
10. Donald Wilber explicitly mentions this issue and mentions about some differences of Ilkhanid era in terms of zone of transition proportions, and especially the elevation of the eight-sided area in the external view (Wilber, 2014: 137).
11. In the past, this dome was covered by an outer covering that was a pyramid with an octagonal base (Godard, 2008: 286).
12. Gholamhossein Memarian, influenced by Pirnia, called these small niches "Potkaneh" (Memarian, 2015: 407).

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